EXPERIMENTS

UPON THE

CONTRACTION OF THE LIQUID VEIN

ISSUING FROM AN ORIFICE,

AND UPON THE

DISTRIBUTION OF THE VELOCITIES

WITHIN IT.

BY

H. BAZIN,

Inspecteur Général des Ponts et Chaussées.

TRANSLATED

FROM MÉMOIRES PRÉSENTÉS PAR DIVERS SAVANTS À L'ACADÉMIE DES SCIENCES DE L'INSTITUT DE FRANCE, TOME XXXII.

BY

JOHN C. TRAUTWINE, JR.,

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FIRST EDITION.

FIRST THOUSAND.

NEW YORK:

JOHN WILEY & SONS.

LONDON: CHAPMAN & HALL, LIMITED.

1896.

ROBERT DRUMMOND, ELECTROTYPER AND PRINTER, NEW YORK.

EXTRACTS

FROM A REPORT OF MESSRS. RESAL, MAURICE LÉVY, SARRAU, AND BOUSSINESQ (*Rapporteur*), COMMITTEE OF THE ACADÉMIE DES SCIENCES, INSTITUT DE FRANCE. *Comptes rendus*, vol. CXVIII.

NOTWITHSTANDING the numerous experiments made since the seventeenth century upon the flow of liquid veins through orifices, there are important matters connected with this phenomenon which still remain undetermined, or so imperfectly known as to give rise to most inexact hypotheses.

Until now we have had no experimental results respecting the pressures exerted in the interior of the vein, or upon the velocities of the separate filaments.

It was therefore highly desirable that delicate observations upon a large scale should be undertaken for the measurement of the pressures and velocities within the issuing vein under considerable heads and with both vertical and horizontal orifices of diverse forms. M. Bazin's memoir contains an account of a large number of just such observations, made at Dijon since 1890, and concluded within the last few months.

The memoir contains an elaborate study of the flow through a vertical rectangular orifice, of the same width as the reservoir itself, and furnished externally with two flat cheek-pieces for preventing the lateral dilation of the vein. These are, so far as we know, the first precise observations made in such

a case, the most important of all from a theoretical point of view, since it is the most elementary, and that to which mathematical analysis can be the most completely applied.

We see, then, that the memoir of M. Bazin realizes in many respects a very marked advance in our knowledge of the important and difficult question of the liquid vein. Your committee has therefore unanimously approved the memoir, and asks of you its insertion in the Recueil des Savants étrangers. In this publication have already appeared, during the present century, the memorable experiments of Poncelet and Lesbros upon the flow through vertical orifices; of Poiseuille upon the flow through capillary tubes; of Darcy, upon the uniform flow in larger pipes; of Darcy and of M. Bazin himself, upon the flow in open channels: a most valuable collection of original documents of the first order in the study of hydrodynamics, and one which will in no wise disparage the new work of M. Bazin.

The conclusions of the report were put to vote and adopted.

TRANSLATOR'S PREFACE.

M. Bazin is perhaps best known to the English-reading public through his investigations of the flow of water in open channels; in which he was associated with M. Darcy, and which formed so important a part of the material employed by Ganguillet and Kutter in the construction of their now world-famous formula.

A few years ago, however, M. Bazin made another important contribution to the literature of hydraulics in the shape of his experiments upon the flow over weirs, the results of which, rendered into English by Mr. Marichal and the present translator, were published in part in the Proceedings of the Engineers' Club of Philadelphia, Volumes VII., IX., and X. In these investigations M. Bazin not only carefully determined the coefficients of discharge under widely varying conditions, but also carried out very delicate measurements for the purpose of determining the shape of the sheet of water, or "nappe," falling over the weir and (by means of the Pitot tube) the velocities and pressures within the sheet itself.

The investigations here presented proceeded upon nearly the same lines as those concerned with flow over weirs, and it is believed that they will be found to constitute an equally important addition to our knowledge of hydraulics.

J. C. T., Jr.

Philadelphia, May, 1896.

AUTHOR'S INTRODUCTION.

PARIS, 15 avril, 1896.

MONSIEUR :—J'ai lu attentivement la traduction, que vous avez bien voulu me communiquer, de mon mémoire sur la contraction des veines liquides et la distribution des vitesses dans leur intérieur.

Cette traduction me parait bien rendre le sens de l'original, et je vous autorise très volontiers a présenter sous cette forme mon travail aux ingénieurs américains.

Votre bien devoué,

H. BAZIN.

Monsieur TRAUTWINE, Engineer, Philadelphia.

DEAR SIR :—I have read attentively the translation, which you have sent me, of my paper on the contraction of the liquid vein and the distribution of the velocities in its interior.

This translation appears to me to render correctly the sense of the original, and I very willingly authorize you to present my work in this form to American engineers.

EXPERIMENTS UPON THE CONTRACTION OF THE LIQUID VEIN.

IN their admirable experiments upon the flow through orifices, Messrs. Poncelet and Lesbros have established a fact which appears at first view to contradict the fundamental principles of hydraulics. In studying the liquid vein issuing from an orifice 0.20 m. square, in thin partition, they found, in 1828, that the mean velocity in the contracted section of the vein was a little greater than the velocity $\sqrt{2gh}$ corresponding to the head h upon the center of that section. Surprised by this anomaly, M. Lesbros considered it necessary to repeat the experiment with the greatest care. This was done in 1834, with but little difference in the results.*

He says: "We must admit that the minimum area of the sections of the vein in planes parallel to that of the orifice is 230.62 square centimeters, so that the coefficient of contraction is $230.62 \div 400$, or, say, 0.577. Now the coefficient deduced from the comparison of the effective and theoretical discharge is 0.602, whence it results that the mean velocity in the contracted section is $\frac{602}{577}$ of that due to the head of the liquid upon the center of the orifice."

From this it appears that the actual velocity is $\frac{1}{23}$ greater

* Expériences hydrauliques sur les lois de l'écoulement de l'eau. Recueil des savants étrangers, vol. III., 1832, and vol. XIII., 1851.

than the theoretical, or $\frac{1}{26}$ greater, if account be taken of the fact that the center of the contracted section is slightly lower than the center of the orifice.

We have repeated the experiments of M. Lesbros with rectangular and circular orifices in vertical and horizontal walls, and, like him, we have found an excess of velocity in the contracted vein when the orifice is in a vertical plane. This, however, we do not find the case with orifices in a horizontal plane.

The orifices with which we have experimented are the following:

1. Orifices in a vertical plane: Square orifice, 0.20 m. square (contraction complete); circular orifice, 0.20 m. in diameter (contraction complete); rectangular orifice, 0.80 m. wide by 0.20 m. high (lateral contraction suppressed).

2. Orifices in a horizontal plane: Circular orifice, 0.20 m. in diameter (contraction complete); circular orifice, 0.10 m. in diameter (contraction complete.)

Having determined the coefficients of discharge by filling a vessel of known capacity, we proceeded to determine the geometrical figure of the vein in order to be able to deduce from the discharge the mean velocity in transverse sections at different distances from the orifice. We have endeavored also to ascertain directly by the use of an instrument analogous to the Darcy tube, the distribution of the velocities through each of the several sections of the issuing vein.

DETERMINATION OF THE COEFFICIENT OF DISCHARGE.

The three vertical orifices were installed successively at the origin of the channel employed in our experiments upon weirs.* Their center was placed 0.60 m. above the bottom of the chan-

* Annales des Ponts et Chaussées, October, 1888.

nel. For the square and circular orifices, the channel retained its width of 2 meters. The sides of the orifices were therefore 0.90 m., from each of the two side walls of the channel.

For the rectangular orifice, in order to suppress the lateral contraction, the width of the channel was reduced to 0.80 m. up-stream from the orifice, which thus occupied its entire width. Two cheek-pieces, A, B, prolonged the walls, for a distance of 0.50 m. down-stream, from the orifice, in order to guide the vein, while preventing its lateral expansion.

As to the horizontal circular orifices, their center was placed at 1 m. from the lateral and terminal walls of the channel, and at 0.70 m. above the ground upon which the vein fell.

The calibration of these orifices was accomplished by following the process employed in calibrating our weirs, that is to say, by filling a part of the channel and noting exactly the time occupied in the filling.* The table on pp. 4, 5 presents a resumé of the results obtained:

The orifices were formed in iron plates 7 mm. in thickness. The edges, carefully finished in a machine, presented a very true, sharp edge.

Let us now compare the values of m given by the foregoing table with those obtained by other experimenters.

Square orifices 0.20 *m. on the side.* The mean of five experiments made under heads between 0.90 m. and 1 m., is $m = 0.6066$, a result corresponding very closely with the value 0.605, adopted by M. Lesbros,† for the same heads.

Rectangular orifice 0.20 *m. high,* 0.80 *m. wide.* Dividing

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* This very simple operation nevertheless requires great care and precautions if we wish to eliminate all causes of error.

For a detailed description of the processes employed, we refer the reader to our first Memoir upon Weirs.

† See the table of coefficients for square orifices given by M. Lesbros, Recueil des savants étrangers, vol. XIII, 1851.

Experiment No.	Head on Centre of Orifice. h	Contents of Channel. Depth.	Contents of Channel. Volume.	Time.	Discharge per Second. q	Coefficient of Discharge. $m = \frac{q}{S\sqrt{2gh}}$
	Meters.	Meters.	Cu. Meters.		Cu. Meters.	
VERTICAL ORIFICE 0.20 M.* SQUARE.						
Area $S = 0.04012$ sq. meter.						
October, 1890. Mean temperature of water, 9° C.						
1	0.9016	0.4416	177.977	29′ 00″	0.10229	0.6062
2	0.9220	0.4332	174.591	28 04	0.10368	0.6076
3	0.9518	0.4676	188.455	29 52	0.10516	0.6066
4	0.9745	0.4505	181.564	28 25	0.10649	0.6071
5	0.9945	0.4592	185.070	28 45	0.10729	0.6054
VERTICAL RECTANGULAR ORIFICE 0.20 M. HIGH, 0.80 M. WIDE,† WITHOUT LATERAL CONTRACTION.						
Area $S = 0.1592$ square meter.						
October, 1890. Mean temperature of water, 13.5° C.						
1	0.8000	0.5400	217.472	9′ 09″	0.39612	0.6280
2	0.8074	0.4760	191.695	8 04	0.39606	0.6251
3	0.8170	0.5257	211.717	8 53	0.39722	0.6233
4	0.8205	0.4600	185.289	7 46	0.39762	0.6225
5	0.8260	0.4994	201.130	8 23	0.39986	0.6240
6	0.8347	0.5535	223.503	9 11	0.40563	0.6296
7	0.8468	0.5078	204.507	8 26	0.40416	0.6228
8	0.8567	0.5201	209.461	8 35	0.40672	0.6231
9	0.8658	0.5215	210.005	8 30	0.41177	0.6276
10	0.8799	0.4970	200.192	8 03	0.41448	0.6266
11	0.8880	0.4670	188.066	7 30	0.41792	0.6289
12	0.8971	0.5010	201.803	8 02	0.41868	0.6270
13	0.9113	0.4827	194.390	7 38	0.42443	0.6306
14	0.9188	0.4556	183.516	7 14	0.42285	0.6255
15	0.9233	0.4550	183.244	7 12	0.42418	0.6260
16	0.9281	0.5550	224 109	8 49	0.42365	0 6237
17	0.9305	0.5190	209.011	8 12	0.42482	0.6246
18	0.9427	0.5238	210.987	8 14	0.42710	0.6239
19	0.9508	0.5022	202.237	7 49	0.43121	0.6271
20	0.9594	0.4807	193.541	7 24	0.43590	0.6312
VERTICAL CIRCULAR ORIFICE 0.20 M. DIAMETER.‡						
Area $S = 0.03132$ square meter.						
April and May, 1890. Mean temperature of water, 11° C.						
1	0.9536	0.4680	188.617	38′ 47″	0.8106	0.5984
2	0.9619	0.4222	170.158	34 51	0.8138	0.5981
3	0.9722	0.4478	180.475	36 47	0.8177	0.5978
4	0.9799	0.3905	157.382	31 59	0.8201	0.5972
5	0.9883	0.4401	177.372	35 53	0.8238	0.5973
6	0.9966	0.4298	173.221	34 54	0.8272	0.5972

* Exactly 0.2003 m.
† Exactly 0.797 m. × 0.1997 m.
‡ Exact mean diameter 0.1997 m.

Experiment No.	Head on Centre of Orifice. h	Contents of Channel. Depth.	Contents of Channel. Volume.	Time.	Discharge per Second.	Coefficient of Discharge. $m=\frac{q}{S\sqrt{2gh}}$
	Meters.	Meters.	Cu. Meters.		Cu. Meters.	

HORIZONTAL CIRCULAR ORIFICE 0.20 M. DIAMETER.*

Area $S = 0.03132$ square meter.

May, 1892. Mean temperature of water 13° C.

1	0.9384	0.4487	88.309	18′ 04″	0.08147	0.6062
2	0.9481	0.4272	84.093	17 07	0.08188	0.6062
3	0.9594	0.4390	86.388	17 35	0.08188	0.6025
4	0.9680	0.4054	79.799	16 11	0.08218	0.6021
5	0.9736	0.4611	90.727	18 17	0.08270	0.6041
6	0.9923	0.4383	86.280	17 16	0.08328	0.6026
7	1.0005	0.4192	82.497	16 27	0.08358	0.6023
8	1.0094	0.4400	86.620	17 11	0.08402	0.6028
9	0.9552	0.4722	92.963	18 53	0.08205	0.6051
10	0.9636	0.4588	90.324	18 16	0.08241	0.6052
11	0.9797	0.4393	86.482	17 26	0.08268	0.6021
12	0.9888	0.4665	91.841	18 23	0.08326	0.6036
13	1.0053	0.4609	90.737	18 05	0.08363	0.6012

Quite frequently, with the horizontal circular orifice, 0.20 m. diameter, an eddy was formed, extending from the orifice to the free surface of the liquid in the channel of approach. It was found possible to prevent the formation of this eddy by allowing a plank to float above the orifice.

As a matter of fact, the eddy did not appear to modify sensibly the discharge, for experiments Nos. 1 to 8 of May, 1892, in which no precaution was taken for its prevention, give the same mean value of m as the five experiments, Nos. 9 to 13, in which, on the contrary, the formation of the eddy was prevented by means of the floating plank.

These eddies caused the formation of a long, narrow tube, drawing in air at the surface of the water and carrying it into the vein, which thus discharged, at the same time, water and air, and lost something of the regularity of its characteristic form.

HORIZONTAL CIRCULAR ORIFICE 0.10 M. DIAMETER.†

Area $S = 0.007886$ square meter.

June and July, 1892. Mean temperature of water 22.5° C.

1	0.9069	0.4850	30.285	25′ 06″	0.02011	0.6046
2	0.9112	0.4849	30.279	25 02	0.02016	0.6047
3	0.9236	0.4932	30.797	25 12	0.02037	0.6068
4	0.9421	0.4922	30.734	25 06	0.02041	0.6021
5	0.9493	0.4750	29.660	23 59	0.02061	0.6056
6	0.9564	0.4784	29.873	24 04	0.02069	0.6057
7	0.9733	0.4939	30.841	24 27	0.02102	0.6100
8	0.9930	0.4928	30.772	24 12	0.02119	0.6087
9	1.0023	0.4911	30.666	24 01	0.02128	0.6085

* Exact mean diameter 0.1997 m.

† Exact mean diameter 0.1002 m.

the series into three groups, following the increase of the heads, we obtain :

Heads between 0.80 m. and 0.85 m.	(7 experiments),	$m = 0.6250$
" " 0 85 m. " 0.90 m.	(5 "),	$m = 0.6266$
" " 0.90 m. " 0.96 m.	(8 "),	$m = 0.6266$

In order to study the influence of variation of the head upon the value of m, we carried out another series of experiments in which the water discharged by the orifice was made to pass over a weir 0.35 m. in height, the coefficient M of which was exactly known.*

This weir having a length of 1.999 m., its discharge under a head H was

$$Q = 1.999 \text{ m.} \times MH\sqrt{2gH};$$

that of the rectangular orifices being, on the other hand,

$$Q = 0.1592 \times m\sqrt{2gh}.$$

The comparison of these two experiments gives immediately

$$m = \frac{1.999}{0.1592} MH\sqrt{\frac{H}{h}}.$$

Examining the last column of the table, where the values of the coefficients are arranged by groups of heads, we see that m first diminishes slightly as the head h increases, but becomes sensibly constant when h exceeds 0.50 m. Its value, for heads between 0.85 m. and 0.96 m., is 0.6307. Direct gauging had given, for the same heads, 0.6266, or $\frac{1}{150}$ less.

But the two processes are not exactly comparable, and the method by the actual measurement of the volume discharged affords a greater guaranty of exactitude for the determination of an absolute value of m.

Our sole object in comparing the discharge of the orifice with that of the weir, the head upon which is always more

* For the table of values of this coefficient, see the Memoir already quoted, Annales des Ponts et Chaussées, October, 1888.

CALIBRATION OF THE RECTANGULAR ORIFICE 0.20 M. HIGH × 0.80 M. WIDE, BY MEANS OF A WEIR WITH FREE NAPPE.*

Height of weir, 0.359 m.; Length of weir, 1.999 m.

October, 1890. Mean temperature of the water, 13° C.

Experiment No.	Head.		Coefficients of Discharge.		Means by Groups.	
	On the Centre of the Orifice. h	On the Weir. H	Weir. M	Orifice. m	$h =$	$m =$
	Meters.	Meters.			Meters.	
1	0.1504	0.1268	0.4401	0.6434		
2	0.1761	0.1332	0.4406	0.6409	0.15 to 0.20	0.6405
3	0.1955	0.1373	0.4410	0.6371		
4	0.2306	0.1443	0.4418	0.6333	0.20 to 0.25	0.6350
5	0.2424	0.1472	0.4421	0.6368		
6	0.2738	0.1531	0.4428	0.6366	0.25 to 0.30	0.6330
7	0.2986	0.1563	0.4432	0.6293		
8	0.3232	0.1610	0.4438	0.6332		
9	0.3492	0.1652	0.4444	0.6340	0.30 to 0.40	0.6324
10	0.3778	0.1692	0.4449	0.6326		
11	0.3984	0.1716	0.4452	0.6296		
12	0.4331	0.1769	0.4459	0.6330		
13	0.4423	0.1776	0.4460	0.6302	0.40 to 0.50	0.6327
14	0.4748	0.1815	0.4465	0.6292		
15	0.4919	0.1853	0.4470	0.6384		
16	0.5249	0.1887	0.4475	0.6358		
17	0.5507	0.1896	0.4476	0.6253	0.50 to 0.60	0.6302
18	0.5779	0.1940	0.4483	0.6327		
19	0.5990	0.1951	0.4485	0.6270		
20	0.6311	0.1985	0.4490	0.6276		
21	0.6436	0.1999	0.4492	0.6283	0.60 to 0.70	0.6294
22	0.6766	0.2032	0.4496	0.6286		
23	0.6924	0.2056	0.4500	0.6330		
24	0.7254	0.2083	0.4504	0.6313		
25	0.7513	0.2097	0.4506	0.6268	0.70 to 0.80	0.6294
26	0.7796	0.2129	0.4510	0.6301		
27	0.7959	0.2141	0.4512	0.6292		
28	0.8245	0.2167	0.4515	0.6298		
29	0.8563	0.2196	0.4519	0.6311	0.80 to 0.90	0.6308
30	0.8784	0.2215	0.4522	0.6316		
31	0.9028	0.2231	0.4525	0.6301		
32	0.9300	0.2253	0.4528	0.6306		
33	0.9575	0.2273	0.4531	0.6301	0.90 to 1.00	0.6304
34	0.9783	0.2297	0.4535	0.6339		
35	1.0059	0.2303	0.4535	0.6274		

* The "nappe" is the sheet of water falling over the weir. It is called "free" when it falls freely through the air without touching the face of the weir.

difficult to determine with precision, was to place in evidence the variation of m relatively to those of the head.

In order to study these more thoroughly, it was, however, necessary to take account of the velocity of approach, which, owing to the large dimensions of the orifice,* was by no means negligible. We adopt, therefore, in the following calculations, the value $m = 0.6266$.

One of the numerous arrangements employed by M. Lesbros is nearly comparable with our rectangular orifice. In that arrangement the square orifice 0.20 m. was placed at the termination of a channel of the same width and the lateral contraction was thus suppressed. The escaping vein was not, as in our experiments, guided by cheek-pieces preventing its lateral expansion, and that expansion, mentioned by M. Lesbros, resulted in a slight increase of the discharge of the orifice. It is this which explains why the coefficient $m = 0.638$, obtained by this skillful experimenter, exceeded by 0.011 that which we have determined.

Circular Orifices.—The coefficient of discharge of circular orifices is nearly constant. We find:

For the vertical orifice 0.20 m. in diameter (mean of 6 experiments), $m = 0.5977$.

For the horizontal orifice 0.20 m. in diameter (mean of 13 experiments), $m = 0.6035$.

For the horizontal orifice 0.10 m. in diameter (mean of 9 experiments), $m = 0.6063$.

We know that this coefficient, varying but slightly from 0.6, does not sensibly increase except in the case of very small orifices or of low heads.

Mr. Hamilton Smith, Jr., discussing numerous experiments

* The influence of this element upon the value of m is never of great importance, since the height $\frac{u^2}{2g}$, which corresponds to the velocity u of approach, does not exceed a few mm.

made by several observers, including himself, has recently published * a table of coefficients applicable to vertical circular orifices. This table gives, for a head of one meter, the following values of m:

Diameter of Orifice.	m.	Diameter of Orifice.	m.
0.006 m	0.627	0.05 m	0.600
0.009 m	0.617	0.10 m	0.599
0.012 m	0.611	0.20 m	0.598
0.015 m	0.606	0.30 m	0.597
0.03 m	0.603		

The value 0.598 is precisely that which we ourselves have obtained for the vertical orifice 0.20 m. in diameter.

As to horizontal orifices, the experimental results are much less numerous. In 1874, Mr. Ellis experimented with orifices 0.30 m. in diameter,† but unfortunately the discharges were determined not by direct measurement of the volume discharged, but by comparison with a weir. The mean of a large number of experiments with heads varying between 0.80 m. and 5.70 m. gives $m = 0.600$. The orifice was submerged. Placing the same orifice vertically and allowing it to discharge into the air, Mr. Ellis obtained, with the same range of heads, $m = 0.592$. Our experiments also give, for a vertical orifice, a value of m slightlv less than that corresponding to a horizontal orifice.

GEOMETRICAL FIGURE OF THE VEIN.

Profile of the Jet.—We first observed the profile described by the vein issuing from vertical orifices, referring the center of its cross-section at each point to horizontal and vertical axes, the origin of co-ordinates being taken at the center of the orifice. This profile is comparable to the parabola described by a projectile subjected to the action of gravity and

* The Flow of Water through Orifices and over Weirs, and through Open Conduits and Pipes, by Hamilton Smith, Jr.; London and New York, 1886.

† Transactions American Society of Civil Engineers, February, 1876.

passing the center of the orifice with a horizontal velocity, $v = \sqrt{2gh}$.

Designating by x the horizontal distance of the projectile from the orifice, and by y its corresponding vertical distance below the center, it is easy to see that the equation of the parabola is $y = \frac{x^2}{4h}$. The curve passing through the centers of the successive sections of the vein differs but little from this parabola, and lies below it, departing from it progressively as the distance from the orifice increases.. Its co-ordinates for square and circular orifices are indicated in the following table:

Square Orifice (h = 0.953 m.).			Circular Orifice (h = 0.990 m.).		
x	y	Ordinate of the Parabola. $y' = \frac{x^2}{4h}$	x	y	Ordinate of the Parabola. $y' = \frac{x^2}{4h}$
0.063	0.001	0.0010	0.08	0.002	0.0016
0.082	0.002	0.0018	0.13	0.006	0.0043
0.104	0.004	0.0028	0.17	0.010	0.0073
0.128	0.006	0.0043	0.235	0.018	0.0139
0.151	0.009	0.0060	0.335	0.035	0.0283
0.175	0.012	0.0080	0.515	0.080	0.0670
0.210	0.017	0.0116			
0.248	0.024	0.0161			
0.302	0.035	0.0239			

The ordinates of the two curves are not proportional, and their ratio approaches unity simultaneously with the increase of their absolute difference.

The following figures show the progressive modifications of liquid veins issuing from a vertical orifice. That issuing from a circular orifice remains regular. Its transverse section, at first exactly circular, is gradually flattened vertically, as is indicated in the successive cross-sections, *ab*, *cd*, and *ef*.

As to the vein issuing from the square orifice, it undergoes a very remarkable change of form, frequently quoted as an ex-

ample of the inversion of the vein. The sections *gh*, *ij*, and *kl* show this gradual transformation, which finally gives to the vein a star-shaped figure, the points of which correspond to the sides of the orifice. This peculiarity explains the singular figure of the longitudinal section of the vein in a vertical plane, following the axis of the channel.

Let us now consider the rectangular orifice. Instead of being circumscribed on all sides, as in the case of the orifices already considered, the vein is of indefinite horizontal extent, and is limited by two surfaces nearly cylindrical, between which we may conceive a mean surface dividing the nappe into two sensibly equal parts. The intersection of this surface with the vertical plane passing through the axis of the channel corresponds to the central curve of the jet determined for square and circular orifices. The upper and lower surfaces of the nappe were observed for the 5 heads h, 0.790 m., 0.836 m., 0.887 m., 0.950 m., and 1.005 m.* Deducing graphically the ordinate y of the mean surface, we recognize that the product hy is sensibly constant for a given horizontal distance x from the orifice, and greater than the quantity $\frac{x^2}{4}$ corresponding to the same product in the parabola $y' = \frac{x^2}{4h}$. We have, in fact,

Distance from Plane of Orifice. x.	Ordinates y of the Central Curve for Heads. $h =$					Products hy for Heads. $h =$					Mean Products hy for the Five Heads.	Value of $\frac{x^2}{4}$.
	0.790	0.836	0.887	0.950	1.005	0.790	0.836	0.887	0.950	1.005		
0.10	0.0078	0.0073	0.0078	0.0072	0.0062	0.0062	0.0061	0.0069	0.0068	0.0062	0.0064	0.0025
0.20	0.0193	0.0184	0.0193	0.0187	0.0164	0.0152	0.0154	0.0171	0.0169	0.0165	0.0162	0.0100
0.30	0.0377	0.0356	0.0354	0.0332	0.0300	0.0298	0.0298	0.0314	0.0315	0.0302	0.0305	0.0225
0.40	0.0628	0.0583	0.0571	0.0529	0.0493	0.0496	0.0487	0.0506	0.0503	0.0495	0.0497	0.0400
0.50	0.0929	0.0868	0.0848	0.0774	0.0733	0.0734	0.0726	0.0752	0.0735	0.0737	0.0737	0.0625
0.60	0.1262	0.1204	0.1169	0.1061	0.1014	0.0997	0.1007	0.1037	0.1008	0.1019	0.1014	0.0900

* The elements of these profiles are given in a special table at the end of the present memoir.

Transverse Section of the Vein.—In order to obtain and reproduce the transverse sections of the veins issuing from square and circular orifices, we surrounded them with an octagonal iron frame placed normally to their axes, and having its perimeter pierced by 24 screws projecting toward its center. These screws were moved little by little until their points touched the surface of the vein. The frame was then placed upon a sheet of paper and the transverse sections were traced and their area measured with great precision.

This process is not applicable to the rectangular orifice, since the section of the vein was represented only by its thickness embraced between the upper and lower surfaces of the nappe. The profiles of these two surfaces were similarly determined by contact with a movable point. This operation, owing to the continual fluctuation of the nappe, was a very delicate one, the nappe being less stable than the contracted veins throughout the entire extent of their perimeter.

We shall first discuss the results of the experiments relative to the orifices with complete contraction, neglecting for the present the rectangular orifice.

The quotient $\frac{m}{\mu}$, which appears in the last column of the table on p. 13, is simply the ratio of the velocity U in the section under consideration, to the velocity $\sqrt{2gh}$, due to the head h upon the center. We have, in fact,

$$U = \frac{q}{\omega} = \frac{mS\sqrt{2gh}}{\omega},$$

and, dividing by $\sqrt{2gh}$ and remarking that $\frac{S}{\omega} = \frac{1}{\mu}$,

$$\frac{U}{\sqrt{2gh}} = \frac{m}{\mu}$$

This ratio, at first less than unity, exceeds this and increases

Distance of Section from Plane of Orifice. x	Ratio of Distance x to Width L of Orifice. $\frac{x}{L}$	Area of Section. ω	Coefficient of Contraction. $\mu = \frac{\omega}{S}$	Ratio of Coefficient of Discharge to Coefficient of Contraction. $\frac{m}{\mu}$
		SQUARE ORIFICE.		
$m = 0.6066$.		$S = 0.04012$ sq. m.		$h = 0.953$ m.
0.063	0.31	0.02586	0.6446	0.941
0.082	0.41	0.02511	0.6259	0.969
0.104	0.52	0.02497	0.6224	0.975
0.128	0.64	0.02467	0.6149	0.986
0.151	0.75	0.02428	0.6052	1.002
0.175	0.87	0.02419	0.6029	1.006
0.210	1.05	0.02395	0.5970	1.016
0.248	1.24	0.02379	0.5930	1.023
0.302	1.51	0.02326	0.5798	1.046
0.350	1.75	0.02320*	0.5783	1.049
		VERTICAL CIRCULAR ORIFICE.		
$m = 0.5977$.		$S = 0.03132$ sq. m.		$h = 0.990$ m.
0.08	0.40	0.01904	0.6079	0.983
0.13	0.65	0.01870	0.5971	1.001
0.17	0.85	0.01864	0.5951	1.004
0.235	1.17	0.01849	0.5904	1.012
0.335	1.67	0.01826	0.5830	1.025
0.515	2.57	0.01782	0.5690	1.050
		HORIZONTAL CIRCULAR ORIFICE 0.20 M. DIAMETER.		
$m = 0.6035$.		$h = 0.975$ m.		$S = 0.03132$ sq. m.
0.075	0.37	0.01880	0.6003	1.005
0.093	0.46	0.01860	0.5939	1.016
0.110	0.55	0.01824	0.5824	1.036
0.128	0.64	0.01796	0.5734	1.053
0.145	0.72	0.01772	0.5658	1.067
0.163	0.81	0.01753	0.5597	1.078
		HORIZONTAL CIRCULAR ORIFICE 0.10 M. DIAMETER.		
$m = 0.6063$.		1° $h = 1$ m.		$S = 0.007886$ sq. m.
0.058	0.58	0.004717	0.5981	1.014
0.088	0.88	0.004632	0.5874	1.032
0.138	1.38	0.004536	0.5752	1.054
0.188	1.88	0.004418	0.5602	1.082
0.288	2.88	0.004231	0.5365	1.130
0.388	3.88	0.004094	0.5191	1.168
0.488	4.88	0.003970	0.5034	1.204
0.588	5.88	0.003870	0.4907	1.236
		2° $h = 0.780$ m.		
0.058	0.58	0.004705	0.5966	1.016
0.088	0.88	0.004584	0.5813	1.043
0.138	1.38	0.004453	0.5647	1.074
0.188	1.88	0.004359	0.5528	1.097

* Mean of 0.0226 and 0.0238.

gradually with the distance from the orifice. With this in view, it becomes necessary to make here a distinction between the axes of vertical and of horizontal orifices. In the first case the two coefficients m and μ become equal when the horizontal distance x from the orifice is about $\frac{7}{10}$ of its width. In the second one that distance is one half less.

In order to compare the velocity U in the section ω with its theoretical value, we must take account of the vertical distance y of the centre of gravity of that section below the centre of the orifice. The ratio which we must consider is therefore not $\frac{U}{\sqrt{2gh}}$, but rather $\frac{U}{\sqrt{2g(h+y)}}$. Hence the foregoing equation may be written $\frac{U}{\sqrt{2g(h+y)}} = \frac{m\sqrt{2gh}}{\mu\sqrt{2g(h+y)}} = \frac{m}{\mu}\sqrt{\frac{h}{h+y}}$.

We have already given the values of y. We may therefore calculate the coefficient of correction $\sqrt{\frac{h}{h+y}}$, and then $\frac{U}{\sqrt{2g(h+y)}}$.

We shall confine ourselves to those sections in which $\frac{m}{\mu}$ exceeds unity. Vertical and horizontal orifices will be considered separately.

VERTICAL ORIFICES.

Distance of Section from Plane of Orifice. x	Distance of Center of Section Below Center of Orifice. y	$\frac{m}{\mu}$	$\sqrt{\frac{h}{h+y}}$	$\frac{U}{\sqrt{2g(h+y)}}$
		SQUARE ORIFICE. $h = 0.953$ m.		
0.151	0.009	1.002	0.9953	0.997
0.175	0.012	1.006	0.9937	1.000
0.210	0.017	1.016	0.9912	1.007
0.248	0.024	1.023	0.9876	1.010
0.302	0.035	1.046	0.9821	1.027
0.350	0.047	1.049	0.9762	1.024

CIRCULAR ORIFICE.

$h = 0.990$ m.

0.13	0.006	1.001	0.9970	0.998
0.17	0.010	1.004	0.9950	0.999
0.235	0.018	1.012	0.9910	1.003
0.335	0.035	1.025	0.9828	1.007
0.515	0.080	1.050	0.9619	1.010

From the foregoing table it follows that with vertical orifices the velocity within the vein, when once contracted, slightly exceeds that due to the head $h + y$. Messrs. Poncelet and Lesbros have already remarked this fact in connection with the square orifice 0.20 m. But the horizontal circular orifices lead to a totally different result.

HORIZONTAL ORIFICES.

Distance of Section from Plane of Orifice. x	Distance of Center of Section Below Center of Orifice. y	$\frac{m}{\mu}$	$\sqrt{\frac{h}{h+y}}$	$\frac{U}{\sqrt{2g(h+y)}}$
HORIZONTAL CIRCULAR ORIFICE 0.20 M. DIAMETER.				
$h = 0.975$.				
0.075	0.075	1.005	0.9636	0.968
0.093	0.093	1.016	0.9555	0.971
0.110	0.110	1.036	0.9480	0.982
0.128	0.128	1.053	0.9402	0.990
0.145	0.145	1.067	0.9330	0.996
0.163	0.163	1.078	0.9256	0.998
HORIZONTAL CIRCULAR ORIFICE 0.10 M. DIAMETER.				
$h = 0.990$.				
0.058	0.058	1.014	0.9720	0.986
0.088	0.088	1.032	0.9583	0.989
0.138	0.138	1.054	0.9369	0.987
0.188	0.188	1.082	0.9168	0.992
0.288	0.288	1.130	0.8802	0.995
0.388	0.388	1.168	0.8476	0.990
0.488	0.488	1.204	0.8185	0.985
0.588	0.588	1.236	0.7921	0.979
$h = 0.790$.				
0.058	0.058	1.016	0.9652	0.981
0.088	0.088	1.043	0.9486	0.989
0.138	0.138	1.074	0.9227	0.991
0.188	0.188	1.097	0.8988	0.986

We thus see that the velocity U does not exceed $\sqrt{2g(h+y)}$ when the orifice is horizontal, that is to say, when the head remains uniform over the entire surface of the orifice, and it is in the inequality of the heads upon the different portions of the orifice that we must seek for an explanation of the anomaly mentioned.

Returning finally to the rectangular orifice without lateral contraction, and performing the same calculations, we have

Distance of Section from Plane of Orifice. x	Ratio of Distance x to Width L of Orifice. $\frac{x}{}$	Area of Section per Linear Meter. ω	Coefficient of Contraction. $\mu = \frac{\omega}{S}$	Ratio of Coefficient of Discharge to Coefficient of Contraction. $\frac{m}{\mu}$	Distance of Center of Section below Center of Orifice. y	$\sqrt{\frac{h}{h+y}}$	$\frac{U}{\sqrt{2g(h+y)}}$
$m = 0.6266$			$S = 0.1997$			$h = 1.000$	
meters							
0.15	0.75	0.1229	0.6154	1.018	0.010	0.9950	1.013
0.20	1.00	0.1204	0.6029	1.039	0.016	0.9921	1.031
0.25	1.25	0.1194	0.5979	1.048	0.023	0.9887	1.036
0.30	1.50	0.1187	0.5944	1.054	0.030	0.9853	1.039
0.35	1.75	0.1185	0.5934	1.056	0.040	0.9806	1.036

The values of $\frac{m}{\mu}$ and of $\frac{U}{\sqrt{2g(h+y)}}$ are greater than those obtained with square and circular orifices. It is necessary, however, to bear constantly in mind that they must undergo a slight reduction. We have admitted that the nappe was perfectly cylindrical, and that in order to estimate ω we must content ourselves with measuring the thickness of the central part of the nappe; but a closer observation has shown that this thickness is not rigorously uniform, and that the vein presents, at a distance of about 0.20 m. on each side of its axis, a slight swelling. We must therefore increase slightly the value of ω, reducing correspondingly the two ratios under consideration. The determination of the correction is rendered very difficult by the motion of the vein. In order to obtain an

approximation of it, we measured, at distances of 0.15 m., 0.16 m., 0.17 m., ... 0.33 m., 0.34 m., and 0.35 m. from the orifice, the thicknesses of the nappe, first in the axis itself, and second at 0.10 m., 0.20 m., and 0.30 m. on each side of that axis. This operation was twice repeated on each side and four times in the axis itself, under a head of 1 meter. Taking the mean of 21 thicknesses thus measured from centimeter to centimeter between the limits $x = 0.15$ m. and $x = 0.35$ m., we obtain the following results:

MEAN THICKNESSES OF VEIN MEASURED ON THE VERTICAL BETWEEN THE LIMITS $x = 0.15$ M. AND $x = 0.35$ M.

(Each of the following figures is the mean of 84 results.)

	Meters.
On the axis	0.1200
0.10 m. to the right and left of the axis	0.1207
0.20 m. " " " " " " " "	0.1226
0.30 m. " " " " " " " "	0.1212

The increase of the thickness at 0.20 m. from the axis is therefore 0.0026 m., or $\frac{1}{46}$. The vein rises a little near the wall of the channel. (See sketch on plate.) The swelling at 0.20 m. appeared in connection with the eddies which sometimes form in the up-stream angles, and was affected also by changes in the head. However this may be, it did not appear that the correction in question exceeded $\frac{1}{100}$, and, even after this reduction, the two ratios, $\frac{m}{\mu}$ and $\frac{U}{\sqrt{2g(h+y)}}$, are still greater than with orifices where the contraction takes place throughout their entire perimeter.

MEASUREMENT OF THE VELOCITIES IN THE INTERIOR OF THE VEINS.

We have endeavored to measure directly the velocities in the interior of the vein by making use of the instrument employed for a similar study in the interior of the nappes in the case of weirs. This instrument, which is simply a particular

form of the Pitot tube, consists of a copper plate 48 mm. wide by 3 mm. thick (Fig. *a*), sharpened at its upper edge and having two brass tubes 2 mm. in interior diameter soldered in a channel formed in its lower edge. These two small tubes have no communication with each other. One of them has its opening at *a* in the up-stream extremity of the plate, and the other at *b* in the lateral face of the plate (Figs. *e* and *f*). The small orifices in which they terminate are 1.5 mm. in diameter and 1 cm. apart.

The plate, introduced into the interior of the vein, does not sensibly modify the flow. It is so placed that its vertical plane is parallel to the axis of the vein, and is held between two iron guides, which enable it to resist the pressure of the fluid. Figs. *a* and *b* show the general arrangement of the apparatus. In Fig. *a*, which refers to the flow from vertical orifices, the plate is horizontal, while in Fig. *b*, which refers to horizontal orifices, it is necessary to turn the plate upward at a right angle so as to present its upper extremity normally to the orifice. The small tube *A*, which we shall call the velocity-tube, opens directly up-stream, and thus receives directly the shock of the liquid vein, while the pressure-tube *B* opens flush with the lateral face of the plate, which is parallel with the direction of flow. It is therefore subject only to the interior pressure of the filament of water passing before its terminal opening. The pressures exerted upon the two orifices *A* and *B* were transmitted by means of the tubes under the plate, and by flexible tubes of lead and caoutchouc, to two vertical glass tubes 8.5 mm. in diameter, open at their upper extremities and placed side by side upon a graduated scale (Fig. *c*). The variations of the level of the water in these tubes thus permits us to observe every modification of the velocities and of the pressures in the different parts of the vein.

We have made 37 experiments with the plate placed in the axis of the vein. They are as follows:

Square orifice.—8 experiments: in the plane of the orifice and at 0.04 m., 0.08 m., 0.12 m., 0.16 m., 0.20 m., 0.25 m., and 0.30 m. down-stream.

Circular vertical orifice.—7 experiments: in the plane of the orifice (under two different heads) and at 0.05 m., 0.09 m., 0.12 m., 0.13 m., and 0.15 m. down-stream.

Rectangular orifice.—6 experiments: in the plane of the orifice and at 0.05 m., 0.10 m., 0.15 m., 0.20 m., and 0.25 m. down-stream.

Circular horizontal orifice, 0.20 *m. in diameter.*—9 experiments in the plane of the orifice and at 0.015 m., 0.035 m., 0.059 m., 0.085 m., 0.112 m., 0.135 m., 0.165 m., and 0.195 m. below the orifice.

Circular horizontal orifice, 0.10 *m. in diameter.*—7 experiments: in the plane of the orifice and at 0.26 m., 0.055 m. (2 experiments), 0.083 m., 0.113 m., and 0.143 m. below it.

The heads varied between 0.95 and 0.99 m., except in the case of one of the two experiments at 0.055 m. from the circular horizontal orifice of 0.10 m. diameter, in which the head was reduced to 0.807 m.

When the instrument just described is placed in the vein, we at once perceive, if the orifice on the velocity-tube is placed normally to the direction of flow, that the water rises in the tube to a level remaining perfectly constant. This level is that of the water up-stream, plus a small head, $\frac{v^2}{2g}$, due to the velocity v of approach, if such velocity exists.*

* This velocity of approach was perceptible only in the case of the rectangular orifice, where $\frac{v^2}{2g}$ attained a value of 0.006 m. or 0.007 m. In the other cases it was negligible.

Designating by A the constant reading in the velocity-tube, we have therefore, for any point whatever in the vein,

$$z + P + \frac{u^2}{2g} = A,$$

where z is the ordinate of that point, u the velocity, and P the pressure.

But when the orifice of the velocity-tube is not normal to the direction of flow, as is the case on the circumference of the vein in the plane of the orifice and in the neighboring sections down-stream, the level A' indicated by the tube becomes less than A.*

At the end of the present memoir will be found detailed tables giving the elements of these experiments. The first column shows the position of the point considered; the second

* From this result we might deduce, if not the exact measure, at least an approximate indication of the inclination α of the axis to the direction of the flow by admitting that the level A' shown by the velocity-tube corresponds to the action of the component $u \cos \alpha$ parallel to the axis, which gives

$$A' = z + P + \frac{u^2 \cos^2 \alpha}{2g},$$

or

$$\frac{u^2}{2g} \cos^2 \alpha = A' - (z + P).$$

If we could turn the instrument so as to present the orifice of the velocity-tube normally to the direction of flow, we should have, neglecting the velocity of approach,

$$h = z + P + \frac{u^2}{2g}$$

or

$$\frac{u^2}{2g} = h - (z + P);$$

from which, dividing the first expression by the second,

$$\cos \alpha = \sqrt{\frac{A' - (z + P)}{h - (z + P)}}.$$

We thus find that the filament situated 20 mm. to one side of the circular orifice would make an angle of about 30° with the axis.

the head h upon the center; the third and the fourth the heads A and B indicated by the two tubes of the instrument. The difference $A - B$ and the value of the pressure P are given in the fifth and sixth columns. The ordinates, z, have been referred to the horizontal plane passing through the center of the orifice, so that the constant elevation A indicated by the velocity-tube is simply the head h upon the center.

Examining the tables we find negative pressures in the sections furthest removed from the orifice, that is to say, pressures inferior to those of the atmosphere. This appears inadmissible, at least in so far as concerns the veins issuing from horizontal orifices where all the filaments converge, describing curves, the convexity of which is turned toward the vertical axis of the vein. These negative pressures cannot in general exist under normal conditions, but must result from the presence of the instrument itself in the vein. It will be readily understood that notwithstanding the thinness of the plate carrying the two tubes the liquid filaments must undergo a certain deviation and describe about the lateral orifice of the pressure-tube a curve whose concavity is turned toward that orifice. From this results a negative pressure, or suction, which reduces the level in the corresponding tube. This effect, which was not observable with the velocities measured by aid of the same instrument in our study of weirs, is rapidly accentuated with the increase of the velocity, which, in the veins issuing from our orifices, exceeded 4 m. per second. The negative pressures appeared first upon the sides of the vein. A little further from the orifice they were found in all parts of the section, but did not exceed a mean of 0.03 m. for vertical orifices and 0.06 m. for horizontal orifices, where the velocity is greater.

When we calculate, by means of the velocities measured at

each point of the vein, the mean velocity for the entire section, and compare it with the value deduced from the discharge obtained in the experiments for calibration, we readily recognize that these negative pressures are really due to the presence of the instrument.

If, in fact, we multiply each element of the surface $d\omega$ by the corresponding velocity u, and then divide the sum $\Sigma u d\omega$ by the total surface, Ω, the mean velocity $U' = \frac{\Sigma u d\omega}{\Omega}$* thus obtained should coincide with the value $U = \frac{Q}{\Omega}$ deduced in calibration; but if, owing to the presence of the instrument, the pressure P becomes negative instead of remaining 0 or positive and very small, the velocities u deduced from the equation $\frac{u^2}{2g} = A - (z + P)$, and, consequently, their mean value U' will be somewhat too great.

We obtain, therefore, by this process, mean velocities greater than those deduced from the discharge, and recognize accordingly the impossibility of attributing negative values to P. We shall first perform this calculation for the circular orifices only.

The calculated velocity U' is therefore too great by about 3.5%, owing to the suction exerted upon the lateral orifice of the instrument, the presence of which causes a slight perturbation of the flow. The error is otherwise not explicable, for

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* This calculation is practicable only in the case of circular horizontal orifices, or of vertical rectangular orifices, these being the only ones where the distribution of the velocities is regular. For the rectangular orifice, the elements $d\omega$ are horizontal rectangles of thickness de. For the circular orifice they are circles of radius r and thickness dr. The mean velocities resulting from this method of calculation are $\frac{\Sigma u de}{E}$ and $\frac{\Sigma 2 u r dr}{R^2}$, where E is the thickness of the rectangular vein and R the radius of the circular vein.

HORIZONTAL CIRCULAR ORIFICE 0.20 M. DIAMETER.

Head. h	Discharge per Second. Q	Distance of Section from Plane of Orifice.	Area of Section. ω	Mean Velocity Deduced from the Discharge. $U = \frac{Q}{\omega}$	Mean Velocity Deduced from Direct Measurements. U'	Ratio. $\frac{U'}{U}$	Remarks
Meters.	Cubic Meters.	Meters.	Square Meters.	Meters per second.	Meters per second.		
0.975	0.08266	0.059	0.01936	4.270	4.414	1 034	A
0.976	0.08270	0.085	0.01869	4.425	4.621	1.044	B
0.975	0.08266	0.112	0.01819	4.544	4.750	1.045	C
0.975	0.08266	0.135	0.01786	4.628	4.820	1.041	D
0.980	0.08288	0.165	0.01750	4.736	4.852	1.024	E
0.969	0.08241	0.195	0.01723	4.783	4.897	1.024	F
HORIZONTAL CIRCULAR ORIFICE 0.10 M. DIAMETER.							
0 807	0.01902	0.055	0.004718	4.031	4.174	1.035	G
0.978	0.02094	0.055	0.004729	4.428	4.578	1.034	H
0.974	0.02090	0.083	0.004646	4.499	4.659	1.036	I
0.976	0.02092	0.113	0.004584	4.564	4.741	1.039	J
0.981	0.02097	0.143	0.004528	4.631	4.805	1.038	K

A. Slight negative pressures at the edges. Mean = — 0.017 m. Maximum pressure, $P = + 0.158$ m.
B. Slight positive pressures in the central region; maximum, $P = 0.035$ m; mean of negative pressures = — 0.035 m.
C. Negative pressures throughout the section. Mean = — 0.055 m.
D. " " " " " " = — 0.067 "
E. " " " " " " = — 0.052 "
F. " " " " " " = — 0.055 "
G. Slight positive pressures at the center. Mean of negative pressures = — 0.022 m.
H. Negative pressures throughout the section. Mean = — 0.033 m.
I. " " " " " " = — 0.043 "
J. " " " " " " = — 0.052 "
K. " " " " " " = — 0.053 "

at a certain distance from the orifice the velocities are perfectly equalized throughout the section of the vein, and there is therefore no reason why pressures less than that of the atmosphere should exist within it.

The rectangular orifice without lateral contraction leads to less definite conclusions, owing to the indeterminateness existing with regard to the true value of Ω, and even in regard to the distribution of the velocities, which last is not exactly the

same throughout the whole extent of the nappe, the nappe itself not being exactly cylindrical.

Distance of Section from Plane of Orifice.	Area of Section per Linear Meter. ω	Deduction of $\frac{U'}{U}$. Head = 0.949 m. Discharge Q = 0.5398 m.^{3} per Linear Meter.			Notes.
		Velocity Deduced from Discharge. $U = \frac{Q}{\omega}$	Velocity Deduced from Direct Measurements. U'	Ratio. $\frac{U'}{U}$	
Meters.	Square Meters.	Meters per Second.	Meters per Second.		
0.15	0.1229	4.392	4.374	0.996	A
0.20	0.1204	4.483	4.402	0.982	B
0.25	0.1194	4.521	4.422	0.978	C

A. Pressure at center = 0. Mean of negative pressures = — 0.014 m.
B. Negative pressures throughout the section. Mean = — 0.020 m.
C. " " " " " " = — 0.019 m.

The values of $\frac{U'}{U}$ instead of being greater than unity, are, on the contrary, a little less, notwithstanding the indication of negative pressures, which, it is true, are less marked than in the preceding cases. It is, however, difficult to draw definite conclusions from them.

DISTRIBUTION OF THE VELOCITIES AND OF THE PRESSURES IN THE PLANE OF THE ORIFICE.

Let us in the first place consider the simple case of a circular horizontal orifice. We have already seen that the water rises in the velocity-tube to a constant elevation, which is that of the water up-stream when the orifice of the tube is presented normally to the direction of flow; in other words, we have generally

$$z + P + \frac{u^2}{2g} = h.$$

In sections of the vein near the orifice, the preceding relation is fully satisfied only near the central portion, the first member of the equation becoming less than h in the neighborhood of the perimeter, owing to the obliquity of the filaments; but we may admit that if we can in this last portion of the vein direct the orifice of the tube normally to the velocities, the instrument will show here, as elsewhere, the constant head h. On the other hand, the plate of the instrument being placed in the vertical plane which divides the orifice symmetrically into two parts, the velocities, necessarily parallel to that plane, have no oblique component which could influence the reading in the pressure-tube by altering the value of P.

The foregoing equation, therefore, remains applicable, and enables us to deduce the value of the velocity u from the observed value of the pressure P. Dividing by h, we may write

$$\frac{z}{h} + \frac{P}{h} + \frac{u^2}{2gh} = 1.$$

In the plane of the horizontal orifice, taken as the plane of reference for the heights, z is 0, and we have, simply,

$$\frac{P}{h} + \frac{u^2}{2gh} = 1.$$

The calculation for the two orifices is as follows: The radius R not being the same, it is best to determine P and u for a series of homologous points at distances of $\frac{1}{10}R$, $\frac{2}{10}R$, $\frac{3}{10}R$.... each side of the center. We thus obtain the results given in the table at the beginning of the next page.

It will be seen that there is, at the center, a minimum velocity about which the other velocities are symmetrically distributed.

CIRCULAR HORIZONTAL ORIFICES.

Distance from Center.	Diameter = 0.20 m. Mean Head, h = 0.974 m.				Diameter = 0.10 m. Mean Head, h = 0.963 m.			
	Value of $\frac{p}{h}$.		Value of $\frac{u}{\sqrt{2gh}}$.		Value of $\frac{p}{h}$.		Value of $\frac{u}{\sqrt{2gh}}$.	
	Above Center.	Below Center.	Above Center.	Below Center.	Above Center.	Below Center.	Above Center.	Below Center.
0 (center)..	0.595		0.636		0.585		0.644	
$0.1R$.......	0.593	0.600	0.638	0.634	0.587	0.575	0.643	0.652
$0.2R$.......	0.580	0.587	0.648	0.642	0.577	0.576	0.650	0.651
$0.3R$.......	0.578	0.572	0.650	0.654	0.576	0.564	0.651	0.660
$0.4R$.......	0.554	0.574	0.667	0.652	0.566	0.567	0.659	0.658
$0.5R$.......	0.549	0.554	0.671	0.668	0.560	0.544	0.663	0.675
$0.6R$.......	0.535	0.542	0.682	0.677	0.526	0.527	0.689	0.688
$0.7R$.......	0.507	0.505	0.702	0.704	....	....	....	....
$0.8R$.......	0.522	0.492	0.691	0.713	....	....	....	....

Three experiments were made at very small distances below the two circular orifices, viz., at 0.015 m., and 0.035 m. for the orifice 0.20 m. in diameter and 0.026 m. for that of 0.10 m. diameter. We will repeat the same calculation for these experiments, remarking that, owing to the verticality of the vein, it is proper to augment by the foregoing quantities the heads h referred to the plane of the orifice itself.

Distance from Center.	Diam. = 0.20 m. Values of $\frac{u}{\sqrt{2gh}}$.						Diam.=0.10 m. Values of $\frac{u}{\sqrt{2gh}}$			
	In the Plane of the Orifice. h = 0.974 m.		0.015 m. below. h = 0.990 m.		0.035 m. below. h = 1.010 m.		In the Plane of the Orifice. h = 0.963 m.		0.026 m. below. h = 0.996 m.	
	Above Center.	Below Center.	Above Center.	Below Center.	Above Center.	Below Center.	Above Center.	Below Center.	Above Center	Below Center
0 (center)	0.636		0.725		0.838		0.644		0.881	
$0.1R$....	0.638	0.634	0.726	0.731	0.853	0.843	0.643	0.652	0.887	0.887
$0.2R$....	0.648	0.642	0.724	0.737	0.864	0.848	0.650	0.651	0.888	0.901
$0.3R$...	0.650	0.654	0.734	0.745	0.863	0.859	0.651	0.660	0.904	0.894
$0.4R$....	0.667	0.652	0.743	0.759	0.904	0.876	0.659	0.658	0.921	0.927
$0.5R$....	0.671	0.668	0.760	0.783	0.900	0.908	0.663	0.675	0.935	0.940
$0.6R$....	0.682	0.677	0.798	0.820	0.948	0.929	0.689	0.688	0.964	0.974
$0.7R$....	0.702	0.704	0.828	0.855	0.981	0.958	....	....	0.984	0.994
$0.8R$....	0.691	0.713	0.898	0.914	1.000	1.000	....	....	1.000	1.000

Examination of the foregoing table shows with what rapidity the velocities vary with the distance from the orifice. For the orifice 0.20 m. diameter the ratio $\frac{u}{\sqrt{2gh}}$ at the center of the vein is 0.84 at a distance of 0.35 m. from the orifice, while in the plane of the orifice itself it is but 0.64. For the orifice 0.10 m. in diameter, a distance 0.026 m. from the orifice, which, relatively to the diameter of the orifice, is greater than in the foregoing case, raised the same ratio from 0.64 to 0.88. At a distance R from the orifice we no longer find a trace of a minimum in the central region, and the velocities are completely equalized throughout the entire cross-section.

The velocities and the pressures vary no less rapidly up-stream from the orifice. We may take account of this by plung- into the basin up-stream, as Lagerjelm did, a vertical tube open at both ends, in such a manner that its lower end is near the plane of the orifice. Lagerjelm's experiment, often quoted, is described in the following terms by Messrs Poncelet and Lesbros;

"M. Rudberg, the learned professor at the University of Stockholm, informed us at the time of his visit to Metz, in 1826, of the result of certain special experiments by M. Lagerjelm which seem to establish this fact,* and which he had occasion to repeat at Paris, in the presence of several members of the Royal Academy of Sciences, notably of M. Ampère. A tube, open at both ends, was plunged vertically above a circular orifice formed in the plane horizontal face of a relatively very large reservoir, in such a manner that its lower extremity

* The fact that the excess of the interior pressure over that of the atmosphere appeared to differ but little from the pressure corresponding to the entire head of the liquid for all the points in the reservoir in the immediate vicinity of the orifice.

was at a little distance on one side or the other of the center of the orifice. Thereupon the liquid was seen to rise vertically in the tube nearly to the upper level of the reservoir, and to maintain practically that level so long as the lower extremity in question was not placed perceptibly below the inner edge of the orifice."

If this fact were exact, that is to say, if the pressure P on the center of the orifice were precisely equal to the head h, the velocity at that point would be zero, and this would be a contradiction of the results which we have just obtained. Desiring to ascertain the reason for this discrepancy, we repeated and completed the experiment of Lagerjelm upon our two orifices 0.20 m. and 0.10 m. in diameter, respectively. For this purpose we placed in the vertical line passing through the center of the orifice a glass tube opened at both ends and moved it up and down in that vertical so as to obtain the pressure not only on the plane of the orifice, but above it, up to the point where there is no longer an appreciable velocity. It was possible even to allow the tube to penetrate a little below the orifice and into the interior of the vein. The fact announced by Lagerjelm was not verified, and a material reduction of the level took place within the tube. A graduated scale attached to the tube and having its divisions visible through the liquid permitted an exact measurement of this reduction. At first we employed a tube 13 mm. in exterior diameter. When its lower extremity A touched the plane of the orifice, the reduction BC of the level, slightly surpassed one half of the head h, hence the pressure AC was less than h; in other words, sensibly less than the pressure obtained in our previous experiments. This difference is easily explained, inasmuch as the mean pressure in the dead space AD, about which the liquid filaments, separated by the tube, are in motion, previous to

their being reunited at D, is less than the pressure upon A. In order to eliminate this source of error, we substituted for the large tube AB a tube EF, tapered at the lower end, and terminating there in a very small orifice, 1 mm. in diameter. The deviation to which the filaments were subjected was thus rendered much less sensible, and in this way we again obtained the value of P already obtained by another process.*

We moved the two tubes vertically away from the plane of the orifice until the reduction of the level became inappreciable. This took place when the elevation of their lower extremity above the orifice was about equal to their diameter. The results are grouped in the two tables on pp. 30, 31. The first of these contains the immediate results of the experiment, that is to say, the pressure P measured in each point defined by its ordinate z above the orifice, and the ratios $\frac{P}{h-z}$ of each pressure to the corresponding head. The second table contains a resumé of the figures given in the first, grouping together those values of $\frac{P}{h-z}$ which correspond, for the two orifices, to homologous points, that is to say, to points whose ordinate z has the same ratio to the diameter $2R$ of the orifice.

An examination of the second table shows in the first place that the results are perfectly in accord for the two orifices, the values of $\frac{P}{h-z}$ being equal for a given value of $\frac{z}{2R}$, and this is the case also with the large tube as well as with the tapering one. The discrepancy between their indications increases as the tube is plunged deeper into the liquid; in other words, as the velocities increase.

* The reduction of the level in the vertical tube increased notably when one of the eddies which we have mentioned was produced.

Height of Lower End of Tube above Plane of Orifice. z	Tapered Tube. Head. h	Tapered Tube. Pressure in the Tube. P	Tapered Tube. Ratio. $\frac{P}{h-z}$.	Cylindrical Tube. Head. h	Cylindrical Tube. Pressure in the Tube. P	Cylindrical Tube. Ratio. $\frac{P}{h-z}$.
Millimeters.	Millimeters.	Millimeters.		Millimeters.	Millimeters	
(1) Orifice 0.20 m. Diameter.						
+ 200	1009	809	1.000	957	756	0.999
+ 160	1006	840	0.993	1020	850	0.988
+ 140	995	845	0.988	997	843	0.984
+ 120	1005	866	0.979	1005	861	0.973
+ 100	1001	871	0.967	998	861	0.959
+ 80	1005	868	0.938	1003	854	0.925
+ 70	1000	863	0.928	1001	843	0.906
+ 60	995	838	0.896	997	821	0.876
+ 50	999	828	0.872	998	792	0.836
+ 45	1001	814	0.851	1000	781	0.818
+ 40	990	800	0.842	997	757	0.791
+ 35	1008	787	0.809	1001	743	0.769
+ 30	1007	769	0.787	1000	719	0.741
+ 25	996	740	0.762	999	684	0.702
+ 20	996	718	0.736	1001	655	0.667
+ 15	1000	686	0.696	1000	610	0.619
+ 10	990	653	0.666	1003	573	0.577
+ 5	980	619	0.635	996	528	0.533
0	999	587	0.588	1006	479	0.476
− 5	998	551	0.549	1002	450	0.447
− 10	1000	503	0.498	998	393	0.390
− 15	1002	481	0.473	1001	355	0.349
− 20	1002	430	0.421	1000	305	0.299
− 25	1008	403	0.390	....	...	....
− 30	995	340	0.332	....	...	....
− 35	985	316	0.310	....	...	....
− 40	1002	280	0.269	....	...	....
− 45	955	250	0.250	....	...	....
− 50	1005	210	0.199	....	...	....
− 55	998	197	0.187	....	...	....
− 60	1003	155	0.146	....	...	....
− 65	1003	115	0.108	....	...	....
(2) Orifice 0.10 m. Diameter.						
+ 100	1022	922	1.000	1034	933	0.999
+ 90	1040	948	0.998	1037	943	0.996
+ 80	1026	938	0.992	1034	947	0.993
+ 70	1040	957	0.987	1036	955	0.989
+ 60	1030	947	0.976	1033	950	0.976
+ 50	1038	954	0.966	1036	947	0.960
+ 40	1032	935	0.943	1033	926	0.933
+ 30	1035	907	0.902	1033	880	0.877
+ 20	1031	849	0.840	1035	801	0.789
+ 15	1032	802	0.789	1034	743	0.729
+ 10	1031	750	0.735	1034	680	0.664
+ 5	1031	690	0.673	1035	602	0.584
+ 2	1032	631	0.613	1036	523	0.506
0	1032	608	0.589	1037	496	0.478
− 2	1032	568	0.549	....	...	....

RÉSUMÉ FOR THE TWO ORIFICES.

Ratio of Height z to Diameter $2R$ of Orifice. $\frac{z}{2R}$.	Value of $\frac{P}{h-z}$.			
	Tapered Tube.		Cylindrical Tube.	
	Diam. = 0.20 m.	Diam. = 0.10 m.	Diam. = 0.20 m.	Diam. = 0.10 m.
+ 1.000	1.000	1.000	0.999	0.999
+ 0.900	....	0.998	....	0.996
+ 0.800	0.993	0.992	0.988	0.993
+ 0.700	0.988	0.987	0.984	0.989
+ 0.600	0.979	0.976	0.973	0.976
+ 0.500	0.967	0.966	0.959	0.960
+ 0.400	0.938	0.943	0.925	0.933
+ 0.350	0.928	....	0.906	....
+ 0.300	0.896	0.903	0.876	0.877
+ 0.250	0.872	....	0.836	....
+ 0.225	0.851	....	0.818	....
+ 0.200	0.842	0.840	0.791	0.789
+ 0.175	0.809	....	0.769	....
+ 0.150	0.787	0.789	0.741	0.729
+ 0.125	0.762	....	0.702	....
+ 0.100	0.736	0.735	0.667	0.664
+ 0.075	0.696	....	0.619	....
+ 0.050	0.666	0.673	0.577	0.584
+ 0.025	0.635	....	0.533	....
+ 0.020	....	0.613	....	0.506
0	0.588	0.589	0.476	0.478
− 0.020	....	0.549	....	....
− 0.025	0.549	....	0.447	....
− 0.050	0.498	....	0.390	....
− 0.075	0.473	....	0.349	....
− 0.100	0.421	....	0.299	....
− 0.125	0.390	....	....	....
− 0.150	0.332	....	....	....
− 0.175	0.310	....	....	....
− 0.200	0.269	....	....	....
− 0.225	0.250	....	....	....
− 0.250	0.199	....	....	....
− 0.275	0.187	....	....	....
− 0.300	0.146	....	....	....
− 0.325	0.108	....	....	....

Considering in particular the very center of the orifice, where z equals 0, we have

	Values of $\frac{P}{h}$.	
	Tapered Tube.	Large Tube.
Orifice 0.20 m. in diameter....	0.588	0.476
" 0.10 m. " "	0.589	0.478

When measuring the pressures we found:

$$\text{Orifice } 0.20 \text{ m. in diameter } \frac{P}{h} = 0.595$$

$$\text{" } 0.10 \text{ m. " " } \frac{P}{h} = 0.585$$

The pressures indicated by the large tube are too small, as we have already indicated. The four other values of $\frac{P}{h}$ are perfectly in accord, and give to that ratio the mean value 0.59. We thus have, for the corresponding velocity, $u = 0.64 \sqrt{2gh}$.

The experiment was repeated with a tapering tube upon two smaller orifices 0.07 m. and 0.05 m. diameter. The reduction of level, as in the case of the two orifices 0.20 m. and 0.10 m. diameter, first became perceptible after rising to a height nearly equal to the diameter. When the head was made to vary between 0.50 m. and 0.90 m., we found that the ratio $\frac{P}{h}$ was constant. Its value was 0.575 for the orifice of 0.07 m. and 0.558 for that of 0.05 m. It thus appeared to diminish slightly with the diameter.

It is not impossible that this diminution may be explained, in part at least, by the presence of the tube, the effect of which becomes more sensible with a small orifice.

The experiment was, however, extremely delicate. The least displacement of the extremity of the tube caused a notable variation of the ratio $\frac{P}{h}$.

The tapering tube having penetrated a little below the plane of the orifice of 0.20 m. diameter, we have, for that region, certain values comparable to those obtained by a direct measurement of pressures.

The accord between the two methods of experimentation is as satisfactory as could be wished.

$\frac{z}{2R}$.	Values of $\frac{P}{h-z}$ obtained.	
	With the Tapering Tube.	At the Time of Measuring the Velocities.
0	0.588	0.595
− 0.025	0.549	....
− 0.050	0.498	....
− 0.075	0.473	0.475
− 0.100	0.421	....
− 0.175	0.310	0.297
− 0.275	0.187	....
− 0.295	....	0.153
− 0.300	0.146	....
− 0.325	0.108	....
− 0.425	....	0.029

Passing to the vertical orifices, the head being no longer constant throughout the surface of the orifice, the discussion of the results obtained with the Pitot tube leads to sensibly different results.

Let us, in the first place, perform the calculation of $\frac{u}{\sqrt{2gh}}$ for the two orifices with complete contraction, giving to z the values $\pm 0.1A$, $\pm 0.2A$ . . . in which A denotes half the height of the opening, say 0.10 m. The sign + corresponds to points situated above the center, and the sign − to those situated below it. (See table on next page.)

There is still, in the plane of the orifice, a minimum velocity $\frac{u}{\sqrt{2gh}}$, the value of which is approximately $u = 0.64\sqrt{2gh}$ for a square orifice, and $u = 0.62\sqrt{2gh}$ for a circular orifice. This minimum is no longer at the center, but a little above it. It disappears rapidly as the distance from the plane of the orifice increases. At 0.08 m. or 0.09 m. from that plane it is hardly perceptible. At 0.10 m. and 0.12 m. it disappears, and the

Ordinate.	Vertical Square Orifice 0.20 m. Square. Mean Head, h = 0.953 m.							Vertical Circular Orifice 0.20 m. Diameter. Mean Head, h = 0.990 m.						
	Ratio.	In the Plane of the Orifice.		0.04 m. from the Orifice.		0.08 m. from the Orifice.		Ratio.	In the Plane of the Orifice.		0.05 m. from the Orifice.		0.09 m. from the Orifice.	
z	$\frac{z}{h}$	$\frac{p}{h}$	$\frac{u}{\sqrt{2gh}}$	$\frac{p}{h}$	$\frac{u}{\sqrt{2gh}}$	$\frac{p}{h}$	$\frac{u}{\sqrt{2gh}}$	$\frac{z}{h}$	$\frac{p}{h}$	$\frac{u}{\sqrt{2gh}}$	$\frac{p}{h}$	$\frac{u}{\sqrt{2gh}}$	$\frac{p}{h}$	$\frac{u}{\sqrt{2gh}}$
− 0.8A..	− 0.084	0.407	0.822	0.016	1.033	0.016	1.033	− 0.081	0.343	0.859	....	....	0.008	1.036
− 0.7A..	− 0.073	0.514	0.750	0.047	1.013	0.031	1.021	− 0.071	0.515	0.745	0 077	0 997	0 023	1.025
− 0.6A..	− 0.063	0.561	0.709	0.105	0.978	0.048	1.007	− 0.061	0.549	0.716	0.142	0.958	0.011	1.020
− 0.5A..	− 0.052	0.598	0.676	0.157	0.946	0.082	0.985	− 0.051	0.590	0.679	0.172	0.937	0.042	1.004
− 0.4A..	− 0.042	0.602	0.662	0.194	0.920	0.092	0.975	− 0.040	0.588	0.673	0.205	0.913	0.046	0.997
− 0.3A..	− 0.031	0.603	0.654	0.257	0.880	0.105	0.963	− 0.030	0.608	0.650	0.242	0.888	0.051	0.990
− 0.2A..	− 0.021	0.604	0.645	0.295	0.852	0.119	0.949	− 0.020	0.611	0.639	0.248	0.879	0.081	0.969
− 0.1A..	− 0.010	0.609	0.634	0.302	0.842	0.112	0.948	− 0.010	0.616	0.628	0.254	0.869	0.079	0.965
0 (center).	0	0.588	0.641	0.308	0.831	0.116	0.941	0	0.602	0.631	0.252	0.864	0.079	0.960
+ 0.1A..	+ 0.010	0.577	0.641	0.310	0.825	0.111	0.938	+ 0.010	0.602	0.623	0.241	0.865	0.065	0.962
+ 0.2A..	+ 0.021	0.565	0.642	0.299	0.826	0.097	0.938	+ 0.020	0.590	0.624	0.230	0.866	0.054	0.962
+ 0.3A..	+ 0.031	0.561	0.638	0.278	0.831	0.088	0.938	+ 0.030	0.582	0.622	0.204	0.874	0.047	0.960
+ 0.4A..	+ 0.042	0.550	0.638	0.243	0.845	0.070	0.943	+ 0.040	0.570	0.625	0.174	0.887	0.033	0.962
+ 0 5A..	+ 0.052	0.533	0.644	0.217	0.854	0.058	0.943	+ 0.051	0.548	0.633	0.143	0.899	0.007	0.971
+ 0.6A..	+ 0.063	0.493	0.666	0.161	0.882	0.038	0.948	+ 0.061	0.529	0.640	0.062	0.936	0	0.969
+ 0.7A..	+ 0.073	0.462	0.680	0.093	0.913	0.014	0.955	+ 0.071	0.489	0.664	0.027	0.949	0	0.964
+ 0.8A..	+ 0.084	0.404	0.715	0.021	0.946	0.007	0.951	+ 0.081	0.456	0.681	....	....	....	....

velocities increase continuously in crossing the vein from above to below.

It now remains only to study the distribution of the velocities of the rectangular orifice without lateral contraction.

Ordinate.	Ratio.	Vertical Rectangular Orifice 0.20 m. High without Lateral Contraction. Mean Head, $h = 0.949$ m.					
		In the Plane of the Orifice.		0.05 m. from the Orifice.		0.10 m. from the Orifice.	
z	$\frac{z}{h}$	$\frac{p}{h}$	$\frac{u}{\sqrt{2gh}}$	$\frac{p}{h}$	$\frac{u}{\sqrt{2gh}}$	$\frac{p}{h}$	$\frac{u}{\sqrt{2gh}}$
$-0.8A$	-0.084	0.290	0.895	....	....	....	....
$-0.7A$	-0.074	0.368	0.844	0.036	1.022	0	1.038
$-0.6A$	-0.063	0.466	0.778	0.090	0.990	0	1.033
$-0.5A$	-0.053	0.491	0.754	0.148	0.954	0	1.028
$-0.4A$	-0.042	0.514	0.731	0.185	0.929	0.013	1.018
$-0.3A$	-0.032	0.528	0.714	0.219	0.905	0.048	0.995
$-0.2A$	-0.021	0.528	0.707	0.243	0.885	0.042	0.993
$-0.1A$	-0.011	0.528	0.699	0.244	0.879	0.050	0.984
0 (center)....	0	0.528	0.692	0.246	0.872	0.041	0.982
$+0.1A$	$+0.011$	0.518	0.690	0.240	0.870	0.033	0.982
$+0.2A$	$+0.021$	0.514	0.687	0.224	0.873	0.021	0.982
$+0.3A$	$+0.032$	0.499	0.689	0.191	0.885	0	0.986
$+0.4A$	$+0.042$	0.470	0.703	0.144	0.906	0	0.985
$+0.5A$	$+0.053$	0.431	0.723	0.092	0.928	0	0.980
$+0.6A$	$+0.063$	0.381	0.750	0.040	0.951	0	0.978
$+0.7A$	$+0.074$	0.309	0.791	0	0.965	....	....
$+0.8A$	$+0.084$	....	....	....	....	....	....

The minimum of the velocities, which, in the orifices with complete contraction, was only from 0.62 to 0.64 $\sqrt{2gh}$, here increases to 0.69 $\sqrt{2gh}$, owing to the suppression of the lateral contraction. It is still perceptible at 0.05 m. from the plane of the orifice, but it disappears at 0.10 m.

RÉSUMÉ OF THE DISCUSSION OF THE EXPERIMENTS.

We shall now endeavor to review in a few words the various results thus far obtained.

In the first place, we remark that in our experiments we

have not observed the generally admitted existence of a contracted vein, if that expression is to be understood in the sense of a *minimum section.* In reality, the vein, after being rapidly contracted upon passing the orifice always continues to contract, much more slowly but constantly, as its distance from the orifice increases.

There is no doubt as to the absence of the minimum of section for the veins issuing from our circular and rectangular orifices. As to the square orifice of 0.20 m. there may be some doubt. Messrs. Poncelet and Lesbros obtained in 1828, for the successive sections of the vein, the following figures:

Distance of Section from the Orifice. x	Area of the Section. ω	Coefficient of Contraction. $\mu = \frac{\omega}{s}$
0.064 m.	0.025205 m.²	0.630
0.110 "	0.024512 "	0.613
0.150 "	0.023746 "	0.594
0.200 "	0.023301 "	0 583
0.250 "	0.023204 "	0.580
0.300 "	0.022506 "	0.563
0.350 "	0.023948 "	0.599
0.400 "	0.024362 "	0.609
0.500 "	0.024427 "	0.611

The value 0.022506 m.² at a distance of 0.30 m., is evidently not exact. M. Lesbros has substituted for it, in consequence of his verifications of 1834, the figure 0.023062 m.² A minimum exists, indeed, in the series, and corresponds to the distance $x = 0.30$ m. Between the orifice and this minimum section, all the values of ω are a little less than that which we ourselves have obtained. The two series are not entirely comparable and the sections do not exactly correspond. The head being about 1.70 m. in the experiments of Messrs. Poncelet and Lesbros, the vein was more elongated than in ours, where the head was only 0.95.

At distances greater than 0.30 m. the section becomes very difficult to measure. The vein, it is true, appears to expand, but, at the same time, it becomes hollow, forming four very sharp edges, and it is indeed uncertain whether the area of the section is really increased.

Operating very carefully with our extreme section at 0.35 m. we arrived at two somewhat different values, 0.0226 m.^{2}, and 0.0238 m.^{2}, both less than those of the foregoing table.

However this may be, the complex form of the vein and its instability render it ill adapted to a theoretical research such as that with which we are occupied.

Examining the distribution of the velocities in the plane of the orifice itself, we find that there exists a minimum. For circular orifices in a horizontal plane this minimum is, of course, at the center. In the case of vertical orifices it is found a little above the center of gravity of the section. We have obtained as the value of this minimum 0.62 to 0.64 $\sqrt{2gh}$ for orifices with complete contraction and 0.69 $\sqrt{2gh}$ for the rectangular orifice with the lateral contraction suppressed.

As we increase the distance from the plane of the orifice, the velocities are rapidly equalized in the vein issuing from a circular orifice, and soon become uniform throughout the entire extent of the transverse section.

This is not the case, however, with orifices in a vertical plane. The minimum existing in the central region soon disappears, but the velocities in the lower part of the vein remain greater than those in the upper part.

The vein diminishes in cross-section as the distance from the orifice increases; the section diminishing and the velocity U increasing by reason of the acceleration due to the fall. If we put $U = K\sqrt{2g(h+y)}$, y representing the fall of the center of the section below that of the orifice, the coefficient K

is slightly less than unity in the case of the horizontal orifice. On the contrary, it exceeds unity for the vertical orifice. In both cases it appears to increase up to a certain distance from the orifice, where it attains a maximum and then diminishes progressively. This maximum would be only a few thousandths less than unity for horizontal circular orifices, but it might attain to 1.03 or 1.04 for vertical orifices, varying with their form and with the head.

The determination of K is very delicate, since it depends upon the measurement of the transverse section, an operation which is rendered difficult by the continual movement in the liquid vein.

With a circular orifice 0.20 m. in diameter, we must, in order to obtain the area of the section within 0.01 of its value, be able to measure its mean diameter with an error not exceeding $\frac{3}{4}$ of a millimeter. The great regularity of the vein permits this degree of approximation, and by measuring it by means of 24 convergent points, we obtain the maximum value of $K = 1.011$.

This measurement is much more difficult when the orifice is square, owing to the singularly complex form of that section. In 1828 M. Lesbros had obtained $K = 1.064$, an exaggerated value, which he expected, after discussing all of the results, to be able to reduce to 1.024. Verification afterward made in 1834 gave 1.038 under a head of 1.71 m., and we ourselves obtained the value $K = 1.027$ with a head of 0.95 m. These differences result from the causes of error inherent in the operation.

The rectangular orifice without lateral contraction appeared at first very favorable to the determination of K, the measurement of the section being reduced to that of the thickness of the nappe. Unfortunately, however, that nappe oscillates in-

cessantly in a manner even more pronounced than that of veins in complete contraction. We have already obtained $K = 1.039$ at 0.30 m. from the orifice, a value probably a little too great.

But, however great the difficulty of an exact determination of K, it is incontestable that that coefficient is greater than unity, with vertical orifices. This conclusion appears at first in contradiction with the fundamental principles of hydraulics.

Concerning a single filament, or a pencil of filaments having equal velocities, we find indeed for K values a little less than unity owing to the internal friction of the liquid. This case is nearly realized in the circular horizontal orifice. The difference, $1 - K$, is always very small, the effect of friction being scarcely appreciable. It would probably be the same for a vertical orifice if the head were very great relatively to its dimensions, so as to equalize the initial velocities, but the question is much more complex when the head is not very great. The velocities in the different filaments are then unequal. Their directions depend upon the configuration of the contour of the orifice, and the formula $U = \sqrt{2g(h + y)}$ is no longer rigorously exact from a theoretical point of view.

In certain portions of the liquid vein the instrument employed gives pressures less than that of the atmosphere. The comparison of these results with the discharges resulting from the direct calibration have shown that the negative pressures were due, at least so far as concerns the circular horizontal orifices, to the suction exerted by the water in motion over the small lateral orifice of the instrument. It is almost impossible, when the velocities are considerable, to eliminate this cause of error. We saw, in repeating the experiment of Lagerjelm, at what point it affected the indications of the vertical tube 13 mm. in external diameter lowered to the center of the

orifice. Although much reduced, it is certainly not completely eliminated by the tapering of the tube. It is not impossible that, for certain special veins, subjected, like that of the square orifice, to considerable deformation, the divergence of certain filaments gives rise to pressures slightly less than that of the atmosphere; but, as we have just said, the experimental demonstration of this fact is very difficult, since we cannot completely protect the instrument used against the perturbing action exerted by neighboring filaments, when these are moving with great velocity.

The experiments which we have described were made, under our direction, by M. Hégly, conducteur des Ponts et Chaussées, whose services were placed at our disposal for the study of the flow over weirs, which we have followed for several years with the assistance of the Minister of Public Works. The knowledge and intelligence of this devoted collaborator have been of the greatest service to us in these delicate researches, which required a high degree of care and precision.

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.

The millimeter is taken as unity. The figure inscribed in the column a (position of the points taken) indicates: 1. For vertical orifices, the ordinate z of the point under consideration referred to a horizontal plane passing through the center of the orifice; 2, for horizontal orifices, the distance measured horizontally from the point under consideration to the up-stream edge of the orifice.

The indication placed at the head of each experiment, and showing the distance of the section from the plane of the orifice, refers to the position of the orifice of the pressure-tube, that of the velocity-tube being 0.01 m. further up-stream.

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$.	Value of $A - B$.
		VERTICAL SQUARE ORIFICE 0.20 M.			
		October and November, 1890. Mean Temperature of the Water, 10.5° C.			
		(1) In the Plane of the Orifice. Mean Head, $h = 0.953$ m.			
− 81	952	800	302	383	498
− 78	952	820	320	398	500
− 75	952	855	390	465	465
− 70	956	865	420	490	445
− 65	953	895	460	525	435
− 60	954	905	475	535	430
− 55	950	922	510	565	412
− 50	956	929	520	570	409
− 45	952	937	525	570	412
− 40	952	940	534	574	406
− 35	955	942	538	573	404
− 30	953	945	545	575	400
− 25	952	947	554	579	393
− 20	952	948	556	576	392
− 15	951	950	558	573	392
− 10	953	953	570	580	383
− 5	951	951	561	566	390
0	952	952	560	560	392
+ 5	952	952	563	558	389
+ 10	952	952	560	550	392
+ 15	953	951	562	547	389
+ 20	953	951	558	538	393
+ 25	954	949	562	537	387
+ 30	954	950	565	535	385
+ 35	953	947	562	527	385
+ 40	953	946	564	524	382
+ 45	952	940	558	513	382
+ 50	953	938	558	508	380
+ 55	952	930	547	492	383
+ 60	953	922	530	470	392
+ 65	953	903	515	450	388
+ 70	950	898	510	440	388
+ 75	950	870	482	407	388
+ 80	952	830	465	385	365
+ 81	953	825	460	379	365

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$.	Value of $A - B$.
(2) 0.04 m. below the Orifice. Mean Head, $h = 0.953$ m.					
− 80	953	951	− 65	15	1016
− 75	951	947	− 50	25	997
− 70	953	950	− 25	45	975
− 65	950	945	+ 10	75	935
− 60	952	948	+ 40	100	908
− 55	953	949	+ 85	140	864
− 50	952	948	+ 100	150	848
− 45	954	951	+ 140	185	811
− 40	952	949	+ 145	185	804
− 35	950	948	+ 185	220	763
− 30	953	953	+ 215	245	738
− 25	953	952	+ 242	267	710
− 20	953	953	+ 261	281	692
− 15	952	952	+ 270	285	682
− 10	953	953	+ 278	288	675
− 5	953	953	+ 290	295	663
0	952	952	+ 294	294	658
+ 5	953	953	+ 298	293	655
+ 10	954	954	+ 305	295	649
+ 15	953	953	+ 300	285	653
+ 20	955	955	+ 305	285	650
+ 25	953	952	+ 295	270	657
+ 30	953	952	+ 295	265	657
+ 35	955	952	+ 295	260	657
+ 40	953	950	+ 272	232	678
+ 45	955	951	+ 259	214	692
+ 50	952	946	+ 257	207	689
+ 55	952	943	+ 217	162	726
+ 60	954	943	+ 213	153	730
+ 65	953	941	+ 170	105	771
+ 70	953	935	+ 159	89	776
+ 75	952	940	+ 121	46	819
+ 80	953	950	+ 100	20	850
+ 82	953	870	+ 90	8	780
(3) 0.08 m. below the Orifice. Mean Head, $h = 0.954$.					
− 82	953	...	− 71	11	....
− 81	953	940	− 65	16	1005
− 78	955	945	− 65	13	1010
− 74	954	950	− 40	34	990
− 70	955	945	− 40	30	985
− 66	952	950	− 18	48	968
− 59	955	942	− 13	46	955
− 51	952	950	+ 28	79	922
− 44	955	943	+ 30	74	913
− 36	953	950	+ 65	101	885
− 29	955	955	+ 70	99	885

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$.	Value of $A - B$.
− 21	953	950	+ 94	115	856
− 14	955	955	+ 95	109	860
− 6	955	951	+ 99	105	852
+ 1	956	956	+ 113	112	843
+ 9	956	951	+ 116	107	835
+ 16	950	950	+ 114	98	836
+ 24	956	951	+ 112	88	839
+ 31	950	950	+ 114	83	836
+ 38	956	951	+ 107	69	844
+ 45	955	958	+ 108	63	850
+ 53	953	951	+ 104	51	847
+ 60	954	957	+ 96	36	861
+ 64	955	950	+ 94	30	856
+ 68	952	950	+ 78	10	872
+ 72	954	944	+ 88	16	856
+ 76	955	936	+ 90	14	846
+ 78	952	...	+ 88	10	...
(4) 0.12 m. below the Orifice. Mean Head, $h = 0.951$.					
− 88	950	850	− 84	+ 4	934
− 85	952	930	− 82	+ 3	1012
− 81	950	940	− 91	− 10	1031
− 78	950	946	− 79	− 1	1025
− 74	952	946	− 78	− 4	1024
− 70	951	949	− 76	− 6	1025
− 66	950	949	− 63	+ 3	1012
− 59	952	952	− 56	+ 3	1008
− 51	950	950	− 40	+ 11	990
− 44	954	954	− 25	+ 19	979
− 36	950	950	− 10	+ 26	960
− 29	951	951	− 2	+ 27	953
− 21	952	952	+ 11	+ 32	941
− 14	950	950	+ 16	+ 30	934
− 6	950	950	+ 28	+ 34	922
+ 1	949	949	+ 34	+ 33	915
+ 9	952	952	+ 40	+ 31	912
+ 16	950	950	+ 45	+ 29	905
+ 24	953	953	+ 52	+ 28	901
+ 31	950	949	+ 60	+ 29	889
+ 38	952	952	+ 60	+ 22	892
+ 45	950	950	+ 66	+ 21	884
+ 53	950	949	+ 62	+ 9	887
+ 60	950	948	+ 70	+ 10	878
+ 64	950	946	+ 67	+ 3	879
+ 68	951	943	+ 65	− 3	878
+ 72	950	935	+ 67	− 5	868
+ 76	953	933	+ 68	− 8	865
+ 77	950	...	+ 68	− 9	...

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube, Of Velocities. A	Level in the Tube, Of Pressures. B	Pressure. $P = B - z$	Value of $A - B$.
		(5) 0.16 m. below the Orifice.	Mean Head, $h = 0.952$.		
− 97	952	...	− 97	0	...
− 96	952	800	− 92	+ 4	892
− 93	952	929	− 85	+ 8	1014
− 89	952	932	− 90	− 1	1022
− 85	950	941	− 87	− 2	1028
− 81	951	945	− 84	− 3	1029
− 74	951	950	− 70	+ 4	1020
− 66	954	952	− 70	− 4	1022
− 59	952	952	− 55	+ 4	1007
− 51	952	952	− 51	0	1003
− 44	953	952	− 35	+ 9	987
− 36	950	951	− 30	+ 6	981
− 29	952	952	− 11	+ 18	963
− 21	952	952	− 9	+ 12	961
− 14	953	953	− 2	+ 12	955
− 6	953	953	+ 2	+ 8	951
+ 1	950	950	+ 13	+ 12	937
+ 9	950	950	+ 18	+ 9	932
+ 16	951	951	+ 25	+ 9	926
+ 24	950	950	+ 35	+ 11	915
+ 31	952	952	+ 43	+ 12	909
+ 38	952	952	+ 46	+ 8	906
+ 45	952	950	+ 55	+ 10	895
+ 53	952	950	+ 55	+ 2	895
+ 60	950	947	+ 64	+ 4	883
+ 64	952	948	+ 70	+ 6	878
+ 68	952	942	+ 65	− 3	877
+ 72	952	937	+ 71	− 1	866
+ 75	950	900	+ 62	− 13	838
+ 76	954	840	+ 71	− 5	769
+ 77	953	...	+ 68	− 9	...
		(6) 0.20 m. below the Orifice.	Mean Head, $h = 0.953$.		
− 108	953	...	− 110	− 2	...
− 107	953	760	− 100	+ 7	860
− 105	955	820	− 94	+ 11	914
− 103	953	900	− 90	+ 13	990
− 100	952	908	− 85	+ 15	993
− 95	950	925	− 92	+ 3	1017
− 88	951	941	− 85	+ 3	1026
− 81	953	948	− 91	− 10	1039
− 73	953	950	− 86	− 13	1036
− 66	952	951	− 85	− 19	1036
− 58	953	952	− 65	− 7	1017
− 51	954	954	− 55	− 4	1009
− 43	952	952	− 40	+ 3	992
− 36	953	953	− 38	− 2	991

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. u	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$.	Value of $A - B$.
− 28	953	953	− 28	0	981
− 21	953	953	− 22	− 1	975
− 13	953	953	− 12	+ 1	965
− 6	953	953	− 9	− 3	962
+ 2	951	951	+ 2	0	949
+ 9	953	953	+ 5	− 4	948
+ 17	953	953	+ 16	− 1	937
+ 24	954	954	+ 18	− 6	936
+ 31	953	952	+ 22	− 9	930
+ 39	952	951	+ 26	− 13	925
+ 46	953	951	+ 38	− 8	913
+ 54	953	949	+ 40	− 14	909
+ 61	952	947	+ 52	− 9	895
+ 69	952	943	+ 55	− 14	888
+ 72	953	925	+ 62	− 10	863
+ 75	953	910	+ 63	− 12	847
+ 76	953	780	+ 65	− 11	715
+ 77	953	...	+ 65	− 12	...
(7) 0.25 m. below the Orifice. Mean Head, $h = 0.953$ m.					
− 126	953	...	− 132	− 6	...
− 125	952	740	− 134	− 9	874
− 118	953	855	− 136	− 18	991
− 111	953	900	− 136	− 25	1036
− 103	952	935	− 132	− 29	1067
− 95	953	942	− 124	− 29	1066
− 88	952	947	− 124	− 36	1071
− 81	955	947	− 125	− 44	1072
− 73	953	950	− 110	− 37	1060
− 66	952	646	− 103	− 37	1049
− 58	955	951	− 85	− 27	1036
− 51	953	951	− 80	− 29	1031
− 43	953	951	− 70	− 27	1021
− 36	952	952	− 68	− 32	1020
− 28	952	952	− 56	− 28	1008
− 21	953	953	− 50	− 29	1003
− 13	953	953	− 42	− 29	995
− 6	953	953	− 38	− 32	991
+ 2	954	954	− 26	− 28	980
+ 9	954	954	− 18	− 27	972
+ 17	952	952	− 12	− 29	964
+ 24	955	955	− 9	− 33	964
+ 31	952	952	+ 3	− 28	949
+ 39	954	953	+ 12	− 27	941
+ 46	953	950	+ 20	− 26	930
+ 54	954	952	+ 33	− 21	919
+ 61	952	947	+ 41	− 20	906

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$	Value of $A - B$.
+ 69	953	946	+ 56	− 13	890
+ 75	954	920	+ 62	− 13	858
+ 77	953	880	+ 65	− 12	815
+ 78	953	...	+ 70	− 8	...
(8) 0.30 m. below the Orifice. Mean Head, h = 0.953.					
− 160	953	...	− 140	+ 20	...
− 156	953	650	− 160	− 4	810
− 148	951	732	− 160	− 12	892
− 141	953	815	− 153	− 12	968
− 133	954	859	− 158	− 25	1017
− 126	953	900	− 156	− 30	1056
− 118	952	918	− 158	− 40	1076
− 111	953	935	− 152	− 41	1087
− 103	953	937	− 150	− 47	1087
− 95	953	944	− 135	− 40	1079
− 88	953	947	− 133	− 45	1080
− 81	955	949	− 120	− 39	1069
− 73	953	950	− 112	− 39	1062
− 66	954	948	− 98	− 32	1046
− 58	950	948	− 91	− 33	1039
− 51	954	952	− 90	− 39	1042
44	954	954	− 77	− 33	1031
− 36	952	952	− 68	− 32	1020
− 29	953	953	− 60	− 31	1013
− 21	952	952	− 48	− 27	1000
− 14	953	953	− 40	− 26	993
− 6	952	952	− 38	− 32	990
+ 2	950	950	− 27	− 29	977
+ 9	954	954	− 26	− 35	980
+ 17	953	953	− 20	− 37	973
+ 24	952	952	− 13	− 37	965
+ 31	953	952	− 8	− 39	960
+ 39	952	952	− 1	− 40	953
+ 46	952	952	+ 4	− 42	948
+ 54	954	950	+ 17	− 37	933
+ 61	954	950	+ 22	− 39	928
+ 69	952	942	+ 42	− 27	900
+ 75	953	930	+ 50	− 25	880
+ 78	953	780	+ 66	− 12	714
+ 80	953	...	+ 66	− 14	...

VERTICAL CIRCULAR ORIFICE 0.20 M. DIAMETER.

June, 1890. Mean Temperature of the Water, 17° C.

(1) In the Plane of the Orifice. Mean Head, h = 0.952 m.

− 76	952	831	340	416	491
− 71	952	856	387	458	469
− 66	953	885	435	501	450
− 61	954	895	458	519	437

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point.	Head on Center of Orifice.	Level in the Tube.		Pressure.	Value of $A - B$.
		Of Velocities. A	Of Pressures. B		
a	h			$P = B - z$	
− 56	952	913	484	540	429
− 51	950	921	494	545	427
− 46	950	932	507	553	425
− 41	952	939	517	558	422
− 36	952	946	531	567	415
− 31	952	946	534	565	412
− 26	951	950	538	564	412
− 21	953	952	551	572	401
− 16	950	951	557	573	394
− 11	952	952	567	578	385
− 6	952	952	561	567	391
− 1	954	954	573	574	381
+ 4	952	952	573	569	379
+ 9	952	953	578	569	375
+ 14	951	951	576	562	375
+ 19	954	953	578	559	375
+ 24	951	950	573	549	377
+ 29	953	951	571	542	380
+ 34	951	949	570	536	379
+ 39	952	948	566	527	382
+ 44	952	941	563	519	378
+ 49	951	936	557	508	379
+ 54	952	928	550	496	378
+ 59	954	926	543	484	383
+ 64	952	910	526	462	384
+ 69	950	898	502	433	396
+ 74	951	878	491	417	387
+ 79	952	848	481	402	367
(2) In the Plane of the Orifice. Mean Head, $h = 0.990$ m.					
− 81	990	828	238	319	590
− 76	990	876	348	424	528
− 71	990	903	443	514	460
− 66	990	919	428	494	491
− 61	992	937	482	543	455
− 56	992	953	494	550	459
− 51	989	958	532	583	426
− 46	992	966	542	588	424
− 41	991	974	541	582	433
− 36	990	978	547	583	431
− 31	990	983	574	605	409
− 26	987	983	562	588	421
− 21	989	987	583	604	404
− 16	988	987	592	608	395
− 11	990	989	601	612	388
− 6	992	992	595	601	397
− 1	989	988	594	595	394
+ 4	991	991	603	599	388
+ 9	990	989	607	598	382

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Poiot. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$	Value of $A - B$.
+ 14	992	993	604	590	389
+ 19	990	989	604	585	385
+ 24	990	990	604	580	386
+ 29	989	987	605	576	382
+ 34	990	988	610	576	378
+ 39	991	984	607	568	377
+ 44	990	974	592	548	382
+ 49	989	969	592	543	377
+ 54	990	973	595	541	378
+ 59	990	962	592	533	370
+ 64	990	940	553	489	387
+ 69	991	938	558	489	380
+ 74	992	905	539	465	366
+ 79	990	903	532	453	371
(3) 0.05 m. below the Orifice.		Mean Head, $h = 0.990$ m.			
− 79	990	989	− 60	19	1049
− 76	987	987	− 12	64	999
− 68	992	992	+ 12	80	980
− 61	990	990	+ 79	140	911
− 53	989	989	+ 98	151	891
− 46	989	989	+ 149	195	840
− 38	990	990	+ 168	206	822
− 31	990	990	+ 209	240	781
− 23	992	992	+ 214	237	778
− 16	989	989	+ 240	256	749
− 8	989	989	+ 242	250	747
− 1	988	988	+ 249	250	739
+ 7	990	990	+ 249	242	741
+ 14	990	990	+ 249	235	741
+ 22	991	991	+ 248	226	743
+ 29	990	990	+ 234	205	756
+ 37	992	992	+ 226	189	766
+ 44	988	988	+ 194	150	794
+ 52	992	992	+ 191	139	801
+ 59	990	990	+ 124	65	866
+ 67	987	987	+ 104	37	883
+ 74	990	990	+ 87	13	903
(4) 0.09 m. below the Orifice.		Mean Head, $h = 0.990$ m.			
− 81	990	990	− 71	+ 10	1061
− 76	990	990	− 74	+ 2	1064
− 68	994	994	− 38	+ 30	1032
− 61	989	989	− 56	+ 5	1045
− 53	989	989	+ 1	+ 54	988
− 46	990	990	− 21	+ 25	1011
− 38	988	988	+ 15	+ 53	973
− 31	990	990	+ 14	+ 45	976
− 23	990	990	+ 60	+ 83	930
− 16	990	990	+ 59	+ 75	931

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$	Value of $A - B$.
− 8	989	989	+ 71	+ 79	918
− 1	990	990	+ 79	+ 80	911
+ 7	991	991	+ 74	+ 67	917
+ 14	991	991	+ 74	+ 60	917
+ 22	990	990	+ 73	+ 51	917
+ 29	990	990	+ 78	+ 49	912
+ 37	990	990	+ 74	+ 37	916
+ 44	989	989	+ 71	+ 27	918
+ 52	989	989	+ 52	0	937
+ 59	990	990	+ 46	− 13	944
+ 67	992	992	+ 51	− 16	941
+ 74	989	989	+ 57	− 17	932
(5) 0.12 m. below the Orifice. Mean Head, $h = 0.951$ m.					
− 78	950	949	− 96	− 18	1045
− 71	950	950	− 105	− 34	1055
− 63	951	951	− 101	− 38	1052
− 56	952	952	− 87	− 31	1039
− 48	950	950	− 89	− 41	1039
− 41	950	950	− 65	− 24	1015
− 33	951	951	− 66	− 33	1017
− 26	950	950	− 46	− 20	996
− 18	951	951	− 41	− 23	992
− 11	951	950	− 30	− 19	980
− 3	950	950	− 29	− 26	979
+ 4	952	950	− 10	− 14	960
+ 12	952	953	− 17	− 29	970
+ 19	952	953	+ 2	− 17	951
+ 27	953	953	0	− 27	953
+ 34	952	953	+ 8	− 26	945
+ 42	955	955	+ 17	− 25	938
+ 49	952	952	+ 17	− 32	935
+ 57	952	953	+ 17	− 40	936
+ 64	950	948	+ 26	− 38	922
+ 72	953	930	+ 49	− 23	881
(6) 0.13 m. below the Orifice. Mean Head, $h = 0.990$ m.					
− 84	989	989	− 99	− 15	1088
− 83	990	990	− 91	− 8	1081
− 75	989	989	− 103	− 28	1092
− 68	990	990	− 91	− 23	1081
− 60	990	990	− 91	− 31	1081
− 53	990	990	− 76	− 23	1066
− 45	990	990	− 71	− 26	1061
− 38	990	990	− 55	− 17	1045
− 30	990	990	− 43	− 13	1033
− 23	990	990	− 43	− 20	1033
− 15	989	989	− 23	− 8	1012
− 8	990	990	− 33	− 25	1023

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$.	Value of $A - B$.
0	989	989	− 12	− 12	1001
+ 7	990	990	− 16	− 23	1006
+ 15	989	989	− 8	− 23	997
+ 22	990	990	+ 4	− 18	986
+ 30	990	990	+ 18	− 12	972
+ 37	990	990	+ 4	− 33	986
+ 45	989	989	+ 18	− 27	971
+ 52	991	991	+ 11	− 41	980
+ 60	990	990	+ 27	− 33	963
+ 67	991	991	+ 49	− 18	942
(7) 0.15 m. below the Orifice. Mean Head, $h = 0.951$.					
− 86	952	952	− 91	− 5	1043
− 78	953	954	− 106	− 28	1060
− 71	949	949	− 107	− 36	1056
− 63	953	954	− 98	− 35	1052
− 56	950	950	− 86	− 30	1036
− 48	953	953	− 82	− 34	1035
− 41	949	949	− 74	− 33	1023
− 33	950	950	− 63	− 30	1013
− 26	951	951	− 50	− 24	1001
− 18	950	950	− 43	− 25	993
− 11	951	951	− 32	− 21	983
− 3	951	951	− 33	− 30	984
+ 4	951	951	− 19	− 23	970
+ 12	949	948	− 17	− 29	965
+ 19	951	951	− 6	− 25	957
+ 27	949	949	− 12	− 39	961
+ 34	951	952	+ 9	− 25	943
+ 42	951	951	+ 6	− 36	945
+ 49	951	951	+ 13	− 36	938
+ 57	950	950	+ 28	− 29	922
+ 64	951	951	+ 40	− 24	911
+ 72	950	950	+ 68	− 4	882
VERTICAL RECTANGULAR ORIFICE 0.20 M. HIGH BY 0.80 M. WIDE. October, 1890. Mean Temperature of the Water, 13.5° C.					
(1) In the Plane of the Orifice. Mean Head, $h = 0.949$ m.					
− 84	949	724	163	247	561
− 81	949	729	177	258	552
− 76	948	792	269	345	523
− 71	950	795	260	331	535
− 66	950	860	355	421	505
− 61	948	880	375	436	505
− 56	949	898	410	466	488
− 51	950	905	410	461	495
− 46	949	925	439	485	486
− 41	948	930	445	486	485
− 36	950	941	459	495	482

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS,—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Of Pressures. B	Pressure. $P = B - z$.	Value of $A - B$.
− 31	949	945	469	500	476
− 26	949	950	480	506	470
− 21	949	952	477	498	475
− 16	949	955	495	511	460
− 11	949	955	488	499	467
− 6	948	954	503	509	451
− 1	949	954	500	501	454
+ 4	948	953	503	499	450
+ 9	948	952	501	492	451
+ 14	949	950	507	493	443
+ 19	949	949	509	490	440
+ 24	950	941	502	478	439
+ 29	948	938	508	479	430
+ 34	949	920	490	456	430
+ 39	949	921	488	449	433
+ 44	950	900	480	436	420
+ 49	949	895	465	416	430
+ 54	949	870	435	381	435
+ 59	950	858	430	371	428
+ 64	948	825	392	328	433
+ 69	950	795	370	301	425
+ 74	950	749	335	261	414
+ 78	948	720	310	232	410
(2) 0.05 m. below the Orifice.			Mean Head, $h = 0.949$ m.		
− 78	949	...	− 75	+ 3	....
− 75	950	943	− 72	+ 3	1015
− 71	950	919	− 42	+ 29	961
− 68	948	915	− 25	+ 43	940
− 64	949	920	+ 5	+ 69	915
− 60	949	925	+ 25	+ 85	900
− 56	950	930	+ 45	+ 101	885
− 49	949	936	+ 98	+ 147	838
− 41	950	943	+ 130	+ 171	813
− 34	947	944	+ 169	+ 203	775
− 26	949	950	+ 188	+ 214	762
− 19	948	955	+ 215	+ 234	740
− 11	949	955	+ 220	+ 231	735
− 4	949	956	+ 232	+ 236	724
+ 4	949	955	+ 235	+ 231	720
+ 11	950	957	+ 238	+ 227	719
+ 19	950	957	+ 235	+ 216	722
+ 26	947	953	+ 220	+ 194	733
+ 33	949	955	+ 205	+ 172	750
+ 40	949	955	+ 177	+ 137	778
+ 47	949	953	+ 150	+ 103	803
+ 55	950	954	+ 117	+ 62	837
+ 59	949	951	+ 102	+ 43	849
+ 62	950	952	+ 91	+ 29	861
+ 66	950	953	+ 68	+ 2	885
+ 70	949	...	+ 72	+ 2	...

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. u	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$.	Value of $A - B$.
(3) 0.10 below the Orifice. Mean Head, $h = 0.949$ m.					
− 71	949	...	− 74	− 3	....
− 67	950	956	76	− 9	1032
− 60	949	956	− 69	− 9	1025
− 52	949	956	− 48	+ 4	1004
− 45	949	956	− 52	− 7	1008
− 37	950	957	− 14	+ 23	971
− 30	949	956	+ 16	+ 46	940
− 22	950	957	+ 16	+ 38	941
− 15	949	957	+ 31	+ 46	926
− 7	950	958	+ 41	+ 48	917
0	949	958	+ 39	+ 39	919
+ 8	950	959	+ 41	+ 33	918
+ 15	949	957	+ 41	+ 26	916
+ 23	949	958	+ 39	+ 16	919
+ 30	950	958	+ 33	+ 3	925
+ 37	949	958	+ 35	− 2	923
+ 44	948	957	+ 37	− 7	920
+ 52	949	957	+ 45	− 7	912
+ 58	950	...	+ 48	− 10	...
(4) 0.15 below the Orifice. Mean Head, $h = 0.949$ m.					
− 75	949	...	− 80	− 5	....
− 71	949	950	− 82	− 11	1032
− 68	949	...	− 81	− 13	....
− 64	949	956	− 86	− 22	1042
− 60	949	...	− 85	− 25	....
− 56	948	954	77	− 21	1031
− 49	950	957	− 77	− 28	1034
− 41	948	956	− 47	− 6	1003
− 34	947	954	− 42	− 8	996
− 26	949	956	− 30	− 4	986
− 19	949	956	− 21	− 2	977
− 11	949	956	− 7	+ 4	963
− 4	949	955	− 3	+ 1	958
+ 4	952	959	+ 1	− 3	958
+ 11	948	955	+ 8	− 3	947
+ 19	948	954	+ 10	− 9	944
+ 25	949	954	+ 13	− 12	941
+ 33	953	960	+ 18	− 15	942
+ 36	949	...	+ 16	− 20	....
+ 40	947	953	+ 21	− 19	932
+ 44	950	...	+ 23	− 21	....
+ 48	951	957	+ 29	− 19	928
+ 51	948	...	+ 25	− 26	....
+ 55	950	...	+ 33	− 22	....

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. u	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$.	Value of $A - B$.
(5) 0.20 m. below the Orifice. Mean Head, $h = 0.949$ m.					
−77	950	...	−95	−18	....
−74	950	950	−98	−24	1048
−71	949	951	−110	−39	1061
−68	949	955	−111	−43	1066
−65	952	956	−102	−37	1058
−61	949	956	−100	−39	1056
−57	950	958	−98	−41	1056
−53	948	956	−80	−27	1036
−46	946	954	−65	−19	1019
−38	949	956	−40	−2	996
−31	949	957	−40	−9	997
−23	949	958	−25	−2	983
−16	948	956	−20	−4	976
−8	949	956	−16	−8	972
−1	948	955	−15	−14	970
+7	948	955	0	−7	955
+14	947	955	0	−14	955
+22	948	955	+10	−12	945
+25	949	958	+2	−23	956
+28	950	958	+10	−18	948
+32	948	956	+5	−27	951
+36	950	957	+18	−18	939
+39	947	952	+20	−19	932
+43	948	945	+20	−23	925
+45	949	...	+22	−23	....
(6) 0.25 m. below the Orifice. Mean Heat, $h = 0.949$ m.					
−83	949	...	−88	−5	....
−80	949	950	−88	−8	1038
−76	948	951	−97	−21	1048
−72	949	955	−100	−28	1055
−68	947	955	−102	−34	1057
−65	949	956	−100	−35	1056
−61	947	955	−90	−29	1045
−56	949	957	−82	−26	1039
−53	947	954	−72	−19	1026
−46	950	958	−62	−16	1020
−39	946	953	−54	−15	1007
−31	950	958	−50	−19	1008
−24	951	960	−30	−6	990
−16	948	958	−35	−19	993
−9	951	959	−16	−7	975
−1	949	958	−16	−15	974
+6	948	958	−8	−14	966
+9	949	957	−2	−11	959
+14	948	956	−2	−16	958
+17	948	956	0	−17	956

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$.	Value of $A - B$.
+ 21	949	957	− 1	− 22	958
+ 24	948	956	+ 5	− 19	951
+ 28	950	958	+ 8	− 20	950
+ 31	947	953	+ 10	− 21	943
+ 35	949	950	+ 8	− 27	942

HORIZONTAL CIRCULAR ORIFICE 0.20 M. DIAMETER.

June, 1892. Mean Temperatnre of the Water, 19°.

(1) In the Plane of the Orifice. Mean Head, $h = 0.974$.

19	970	835	450	450	385
20	975	850	509	509	341
23	982	860	468	468	392
28	982	875	486	486	389
33	973	895	510	510	385
38	960	890	509	509	381
48	975	942	539	539	403
53	973	935	530	530	405
58	975	955	535	535	420
69	985	970	570	570	400
74	975	965	562	562	403
79	965	965	558	558	407
89	975	975	578	578	397
94	975	975	578	578	397
99	968	968	575	575	393
104	975	975	584	584	391
109	975	975	585	585	390
114	972	972	578	578	394
119	975	975	575	575	400
129	968	968	550	550	418
135	975	975	572	572	403
140	975	975	560	560	415
150	975	975	540	540	435
155	980	967	553	553	414
160	975	958	528	528	430
171	975	943	488	488	455
176	972	900	502	502	398
179	975	880	490	490	390
181	978	900	470	470	430

(2) 0.15 m. below the Orifice. Mean Head, $h = 0.975$ m.

15	976	775	120	135	655
18	975	820	143	158	677
23	975	855	225	240	630
28	974	885	270	285	615
33	975	900	330	345	570
38	973	908	340	355	568
44	975	925	355	370	570
49	970	940	395	410	545
54	975	940	430	445	510

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$.	Value of $A - B$.
64	975	965	426	441	539
74	978	974	452	467	522
84	971	968	455	470	513
94	980	980	453	468	527
105	975	975	458	473	517
115	980	980	435	450	545
125	974	974	440	455	534
135	980	980	409	424	571
145	975	968	404	419	564
150	975	968	368	383	600
155	976	965	380	395	585
160	975	950	310	325	640
165	975	942	310	325	632
170	967	910	250	265	660
176	979	880	182	197	698
179	975	885	170	185	715
181	970	865	126	141	739
183	970	850	103	118	747
(3) 0.035 m. below the Orifice.			Mean Head, $h = 0.975$ m.		
14	975	969	− 44	− 9	1013
17	976	959	− 44	− 9	1003
20	974	949	− 34	+ 1	983
25	974	947	− 4	+ 31	951
30	977	950	+ 4	+ 39	946
40	972	972	+ 68	+ 103	904
50	977	969	+ 158	+ 193	811
60	975	975	+ 149	+ 184	826
70	977	974	+ 220	+ 255	754
80	974	974	+ 221	+ 256	753
90	978	978	+ 241	+ 276	737
100	971	971	+ 264	+ 299	707
110	975	977	+ 259	+ 294	718
120	971	972	+ 248	+ 283	724
130	974	974	+ 229	+ 264	745
140	972	972	+ 199	+ 234	773
150	982	982	+ 144	+ 179	838
160	974	967	+ 103	+ 138	864
170	974	968	+ 49	+ 84	919
175	979	965	− 19	+ 16	984
180	977	969	− 43	− 8	1012
183	974	971	− 40	− 5	1011
(4) 0.059 m. below the Orifice.			Mean Head, $h = 0.975$ m.		
18	977	...	− 65	− 6	....
20	977	977	− 71	− 12	1048
21	977	977	− 75	− 16	1052
23	981	981	− 83	− 24	1064
28	973	973	− 49	+ 10	1022

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$.	Value of $A - B$.
33	971	971	− 44	+ 15	1015
43	977	974	− 21	+ 38	995
48	974	974	+ 24	+ 83	950
53	974	974	+ 7	+ 66	967
63	972	973	+ 44	+ 103	929
68	974	974	+ 57	+ 116	917
73	969	969	+ 31	+ 90	938
83	979	979	+ 77	+ 136	902
88	976	976	+ 84	+ 143	892
93	974	974	+ 106	+ 165	868
99	971	971	+ 99	+ 158	872
103	974	978	+ 93	+ 152	885
108	974	974	+ 61	+ 120	913
113	969	972	+ 57	+ 116	915
123	979	979	+ 59	+ 118	920
128	974	974	+ 24	+ 83	950
133	974	974	+ 29	+ 88	945
143	977	977	− 16	+ 43	993
148	974	974	− 1	+ 58	975
153	974	974	− 11	+ 48	985
163	974	974	− 93	− 34	1067
168	974	974	− 39	+ 20	1013
173	974	974	− 93	− 34	1067
176	978	978	− 82	− 23	1060
178	974	974	− 76	− 17	1050
179	974	974	− 63	− 4	1037
181	974	971	− 63	− 4	1034
(5) 0.085 m. below the Orifice. Mean Head, $h = 0.976$ m.					
22	974	974	− 94	− 9	1068
27	974	974	− 133	− 48	1107
32	978	979	− 141	− 56	1120
42	984	984	− 151	− 66	1135
52	984	984	− 119	− 34	1103
62	979	979	− 106	− 21	1085
72	974	974	− 80	+ 5	1054
82	977	977	− 64	+ 21	1041
92	974	975	− 49	+ 36	1024
100	974	974	− 54	+ 31	1028
102	975	975	− 44	+ 41	1019
112	975	977	− 61	+ 24	1038
122	975	975	− 62	+ 23	1037
132	979	982	− 63	+ 22	1045
142	972	972	− 99	− 14	1071
152	983	983	− 93	− 8	1076
162	974	974	− 146	− 61	1120
172	959	964	− 136	− 51	1100
177	975	975	− 117	− 32	1092
179	976	...	− 101	− 16	....

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$	Value of $A - B$.
(6) 0.112 m. below the Orifice. Mean Head, $h = 0.975$ m.					
24	982	...	− 137	− 25	....
26	979	977	− 144	− 32	1121
31	974	974	− 171	− 59	1145
41	979	979	− 178	− 66	1157
46	972	972	− 177	− 65	1149
51	973	973	− 184	− 72	1157
61	979	979	− 178	− 66	1157
71	969	969	− 167	− 55	1136
81	977	977	− 164	− 52	1141
91	970	970	− 143	− 31	1113
96	974	974	− 166	− 54	1140
101	974	974	− 169	− 57	1143
106	975	975	− 151	− 39	1126
111	973	973	− 163	− 51	1136
121	974	974	− 162	− 50	1136
131	971	971	− 172	− 60	1143
141	971	971	− 174	− 62	1145
151	974	974	− 203	− 91	1177
156	974	974	− 200	− 88	1174
161	971	971	− 184	− 72	1155
171	975	975	− 183	− 71	1158
174	977	977	− 157	− 45	1134
176	976	976	− 161	− 49	1137
178	977	977	− 131	− 19	1108
(7) 0.135 m. below the Orifice. Mean Head, $h = 0.975$ m.					
24	974	...	− 151	− 16	....
25	974	959	− 164	− 29	1123
27	979	979	− 169	− 34	1148
32	978	978	− 206	− 71	1184
42	981	981	− 207	− 72	1188
47	969	969	− 217	− 82	1186
52	979	979	− 221	− 86	1200
62	979	979	− 201	− 66	1180
72	972	972	− 228	− 93	1200
82	971	971	− 196	− 61	1167
92	969	969	− 191	− 56	1160
97	974	974	− 191	− 56	1165
100	974	974	− 197	− 62	1171
102	977	977	− 221	− 86	1198
112	975	975	− 224	− 89	1199
122	979	979	− 236	− 101	1215
132	975	975	− 228	− 93	1203
142	969	969	− 233	− 98	1202
147	969	969	− 243	− 108	1212

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$	Value of $A - B$.
152	976	976	− 239	− 104	1215
162	971	971	− 198	− 63	1169
172	976	976	− 191	− 56	1167
175	977	977	− 151	− 16	1128
177	974	...	− 151	− 16	....
(8) 0.165 m. below the Orifice.			Mean Head, $h = 0.980$ m.		
25	970	...	− 175	− 10	....
27	980	980	− 178	− 13	1158
30	968	968	− 203	− 38	1171
35	980	980	− 185	− 20	1165
40	970	970	− 225	− 60	1195
50	979	979	− 225	− 60	1204
55	978	978	− 228	− 63	1206
60	986	986	− 230	− 65	1216
70	980	980	− 230	− 65	1210
75	978	978	− 253	− 88	1231
80	976	976	− 222	− 57	1198
90	982	982	− 208	− 43	1190
95	985	985	− 235	− 70	1220
100	980	980	− 190	− 25	1170
101	975	975	− 193	− 28	1168
105	985	985	− 240	− 75	1225
110	975	975	− 250	− 85	1225
115	987	987	− 222	− 57	1209
125	988	988	− 232	− 67	1220
130	975	975	− 240	− 75	1215
135	988	988	− 210	− 45	1198
145	988	988	− 220	− 55	1208
150	978	978	− 230	− 65	1208
155	985	985	− 235	− 70	1220
165	985	985	− 220	− 55	1205
175	980	970	− 200	− 35	1170
177	985	...	− 178	− 13	....
(9) 0.195 m. below the Orifice.			Mean Head, $h = 0.969$ m.		
24	965	965	− 205	− 10	1170
29	965	965	− 248	− 53	1213
34	950	950	− 273	− 78	1223
39	960	960	− 286	− 91	1246
44	960	960	− 302	− 107	1262
44	962	962	− 247	− 52	1209
49	960	960	− 285	− 90	1245
54	972	972	− 283	− 88	1255
59	982	982	− 273	− 78	1255
64	945	945	− 261	− 66	1206
64	961	961	− 265	− 70	1226
69	963	963	− 271	− 76	1234
74	968	968	− 255	− 60	1223

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$	Value of $A - B$.
79	960	960	− 254	− 59	1214
84	987	987	− 252	− 57	1239
89	978	978	− 269	− 74	1247
94	960	960	− 222	− 27	1182
99	973	975	− 271	− 76	1246
104	990	990	− 200	− 5	1190
109	975	975	− 256	− 61	1231
114	950	950	− 220	− 25	1170
119	965	965	− 266	− 71	1231
124	975	975	− 220	− 25	1195
129	975	975	− 261	− 66	1236
134	995	995	− 222	− 27	1217
139	966	966	− 248	− 53	1214
139	970	970	− 240	− 45	1210
144	950	950	− 230	− 35	1180
149	963	963	− 228	− 33	1191
154	988	988	− 248	− 53	1236
154	965	965	− 220	− 25	1185
159	968	968	− 210	− 15	1178
164	996	996	− 257	− 62	1253
169	965	965	− 255	− 60	1220
174	995	...	− 215	− 20	....

HORIZONTAL CIRCULAR ORIFICE, 0.10 M. DIAMETER.

July, 1892. Mean temperature of Water, 24° C., except for the Experiments made in the Plane of the Orifice which were made in April, 1894; Temperature of Water, 14° C.

(1) In the Plane of the Orifice. Mean Head, $h = 0.963$ m.

20	965	923	508	508	415
25	955	930	535	535	395
30	963	950	545	545	405
35	990	982	570	570	412
40	953	950	550	550	400
45	968	967	568	568	399
50	968	967	565	565	402
50	943	943	553	553	390
50	940	938	550	550	388
50	970	970	566	566	404
55	982	984	565	565	419
60	966	961	556	556	405
65	972	973	548	548	425
70	967	958	548	548	410
75	941	935	512	512	423
80	972	933	512	512	421

(2) 0.026 m. below the Orifice. Mean Head, $h = 0.970$ m.

6	975	...	− 28	− 2	...
7	965	955	− 30	− 4	985
11	970	955	− 28	− 2	983
16	987	963	+ 14	+ 40	949

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point. a	Head on Center of Orifice. h	Level in the Tube. Of Velocities. A	Level in the Tube. Of Pressures. B	Pressure. $P = B - z$	Value of $A - B$.
21	960	940	+ 52	+ 78	888
26	990	980	+ 113	+ 139	867
31	970	965	+ 129	+ 155	836
36	981	979	+ 165	+ 191	814
41	988	988	+ 195	+ 221	793
46	978	978	+ 187	+ 213	791
50	945	945	+ 185	+ 211	760
51	976	976	+ 205	+ 231	771
51	974	974	+ 200	+ 226	774
56	975	975	+ 184	+ 210	791
61	960	957	+ 155	+ 181	802
66	974	970	+ 180	+ 206	790
71	972	967	+ 100	+ 126	867
75	975	968	+ 90	+ 116	878
76	958	948	+ 51	+ 77	897
81	955	943	+ 20	+ 46	923
86	955	935	− 23	+ 3	958
89	960	950	− 27	− 1	977
91	960	950	− 30	− 4	980
94	975	...	− 30	− 4	...
(3) 0.055 m. below the Orifice. Mean Head, $h = 0.807$ m.					
10	808	...	− 70	− 15	...
12	806	775	− 68	− 13	843
15	812	812	− 84	− 29	896
20	803	803	− 95	− 40	898
25	812	812	− 82	− 27	894
30	800	800	− 68	− 13	868
35	810	810	− 58	− 3	868
40	806	806	− 60	− 5	866
45	812	812	− 30	+ 25	842
50	800	800	− 42	+ 13	842
50	808	808	− 58	− 3	866
50	812	812	− 45	+ 10	857
55	815	815	− 50	+ 5	865
60	800	800	− 82	− 27	882
65	818	818	− 58	− 3	876
70	797	797	− 90	− 35	887
75	815	815	− 85	− 30	900
80	798	798	− 92	− 37	890
85	809	809	− 98	− 43	907
88	810	807	− 92	− 37	899
90	800	700	− 75	− 20	775
(4) 0.055 m. below the Orifice. Mean Head, $h = 0.978$ m.					
10	965	...	− 58	− 3	....
11	970	940	− 70	− 15	1010
16	975	975	− 93	− 38	1068
21	982	982	− 108	− 53	1090

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURES IN THE INTERIOR OF THE LIQUID VEINS.—*Continued.*

Position of Point.	Head on Center of Orifice.	Level in the Tube.		Pressure.	Value of $A - B$.
		Of Velocities.	Of Pressures.		
u	h	A	B	$P = B - z$	
26	970	970	− 102	− 47	1072
31	978	978	− 102	− 47	1080
36	965	965	− 114	− 59	1079
41	990	990	− 102	− 47	1092
46	952	952	− 100	− 45	1052
46	992	992	− 105	− 50	1097
50	975	975	− 68	− 13	1043
50	982	982	− 58	− 3	1040
51	1000	1000	− 100	− 45	1100
56	965	965	− 108	− 53	1073
61	1000	1000	− 85	− 30	1085
66	970	970	− 80	− 25	1050
71	995	995	− 92	− 37	1087
76	972	972	− 75	− 20	1047
81	980	980	− 85	− 30	1065
86	980	980	− 85	− 30	1065
88	975	975	− 79	− 24	1054
90	980	980	− 65	− 10	1045
(5) 0.083 m. below the Orifice. Mean Head, h = 0.974 m.					
11	990	...	− 95	− 12	....
12	980	950	− 95	− 12	1045
14	965	962	− 105	− 22	1067
16	965	965	− 125	− 42	1090
18	995	995	− 132	− 49	1127
21	960	960	− 142	− 59	1102
26	955	955	− 140	− 57	1095
31	950	950	− 130	− 47	1080
36	960	960	− 122	− 39	1082
41	975	975	− 115	− 32	1090
46	980	980	− 112	− 29	1092
50	982	982	− 135	− 52	1117
51	962	962	− 120	− 37	1082
51	965	965	− 130	− 47	1095
56	972	972	− 89	− 6	1061
61	975	975	− 130	− 47	1105
66	990	990	− 118	− 35	1108
71	973	973	− 155	− 72	1128
76	967	967	− 148	− 65	1115
76	990	990	− 150	− 67	1140
81	975	975	− 150	− 67	1125
81	990	990	− 170	− 87	1160
86	995	995	− 125	− 42	1120
89	975	...	− 90	− 7	....
(6) 0.113 m. below the Orifice. Mean Head, h = 0.976 m.					
12	976	...	− 130	− 17	....
13	975	900	− 138	− 25	1038
16	976	976	− 155	− 42	1131

DETERMINATION OF THE VELOCITIES AND OF THE PRESSURE IN THE INTERIOR OF THE LIQUID VEINS.—*Concluded.*

Position of Point.	Head on Center of Orifice.	Level in the Tube.		Pressure.	Value of $A - B$.
		Of Velocities.	Of Pressures.		
u	h	A	B	$P = B - z$	
21	978	978	− 175	− 62	1153
26	971	971	− 185	− 72	1156
31	977	977	− 180	− 67	1157
36	972	972	− 178	− 65	1150
41	976	976	− 156	− 43	1132
46	972	972	− 175	− 62	1147
50	970	970	− 165	− 52	1135
51	978	978	− 170	− 57	1148
56	975	975	− 163	− 50	1138
61	976	976	− 185	− 72	1161
66	978	978	− 165	− 52	1143
71	976	975	− 200	− 87	1175
76	980	980	− 190	− 77	1170
81	977	975	− 180	− 67	1155
86	976	976	− 150	− 37	1126
88	978	800	− 128	− 15	928
89	977	...	− 125	− 12	....
(7) 0.143 m. below the Orifice. Mean Head, $h = 0.981$ m.					
11	996	800	− 160	− 17	960
12	973	973	− 173	− 30	1146
12	1000	980	− 170	− 27	1150
16	990	990	− 190	− 47	1180
21	973	973	− 192	− 49	1165
26	986	986	− 212	− 69	1198
31	968	968	− 185	− 42	1153
36	1000	1000	− 205	− 62	1205
36	974	974	− 212	− 69	1186
41	980	980	− 195	− 52	1175
46	986	986	− 215	− 72	1201
50	973	973	− 210	− 67	1183
50	980	980	− 220	− 77	1200
50	990	990	− 211	− 68	1201
11	972	972	− 190	− 47	1162
56	1000	1000	− 209	− 66	1209
56	973	973	− 210	− 67	1183
61	975	975	− 210	− 67	1185
66	985	985	− 206	− 63	1191
71	976	975	− 218	− 75	1193
76	980	980	− 210	− 67	1190
81	972	972	− 208	− 65	1180
84	972	972	− 185	− 42	1157
86	980	930	− 170	− 27	1100
88	972	900	− 170	− 27	1070
89	972	...	− 150	− 7	....

PROFILES OF THE VEIN FOR THE RECTANGULAR ORIFICE.

0.20 m. high by 0.80 m. wide.

Measurements in Millimeters.

(1) Lower Surface.

The point o is 19 millimeters from the orifice for the heads 787 and 888, and 23 millimeters for the heads 836, 950, and 1006.

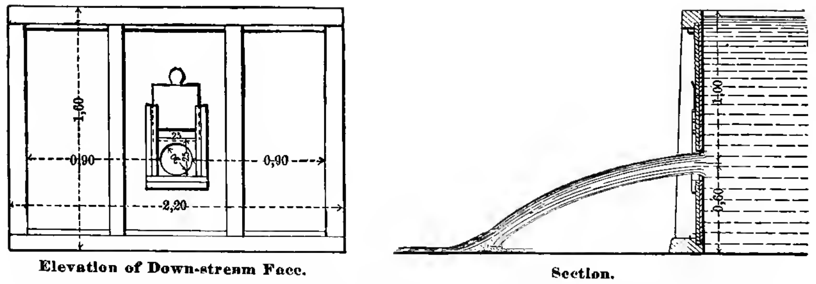
Abscissas or Distances from the Point o.	Ordinates.				
	h = 787.	h = 836.	h = 888.	h = 950.	h = 1006.
0	20.3	19.9	20.0	20.4	20.1
10	22.7	21.8	21.8	23.5	23.5
20	24.5	25.3	24.4	25.2	25.3
30	25.7	26.8	25.5	26.6	27.8
40	27.3	28.3	28.0	28.2	28.8
50	27.9	29.3	27.5	29.5	28.8
60	28.6	28.3	29.0	29.3	29.5
70	27.5	29.1	29.3	29.3	29.9
80	27.8	29.3	28.9	29.3	30.8
90	26.9	27.0	28.5	29.0	30.1
100	27.6	27.5	27.8	27.9	30.0
110	26.5	26.8	27.5	27.2	29.1
120	25 2	27.3	27.4	26.8	29.9
130	26.0	26.0	26.3	26.2	27.3
140	23.2	26.0	24.4	25.1	25.9
150	23.5	23.8	24.4	25.1	25.5
160	20.5	23.0	24.3	23.5	23.4
170	19.5	23.0	23.0	21.8	22.3
180	19.0	19.5	21.5	21.3	21.5
190	19.0	18.3	21.5	20.1	21.5
200	....	18.3	....	18.3	....
210	14.5	17.5	17.5	19.1	19.8
230	12.0	13.3	14.9	16.3	16.8
250	9.3	9.8	11.0	10.8	13.3
270	3.5	2.8	8.0	7.4	10.8
290	0.5	0.8	4.0	6.0	7.1
310	....	− 1.0	....	1.8	5.3
320	− 8.3	....	− 2.0	....	....
330	....	− 4.7	....	− 3.0	0.8
350	− 16.5	....	− 8.9	....	− 3.7
360	....	− 11.7	....	− 11.7	....
370	....	....	....	....	− 8.4
390	− 25.5	− 22.7	− 18.5	− 17.9	− 13.8
420	....	....	....	− 23.9	− 20.5
430	....	− 34.7	− 25.2	....	....
450	....	....	....	− 31.7	− 24.7
490	....	....	....	− 48.0	− 33.7
530	....	....	....	− 54.6	− 48.7

PROFILES OF VEIN FOR RECTANGULAR ORIFICE.—*Concluded.*

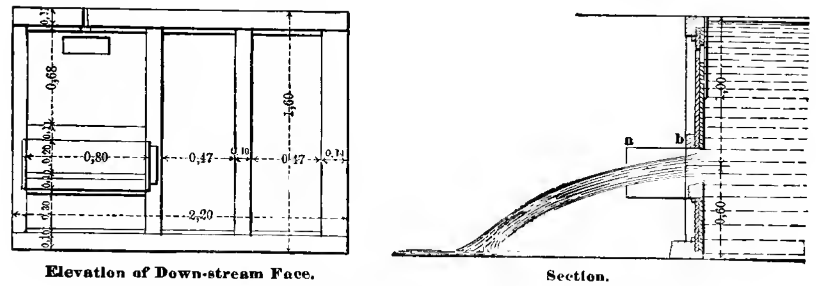
(2) Upper Surface.

The point o is 19 millimeters from the Orifice for the heads 792 and 836, and 23 millimeters for the heads 885, 949, and 1004.

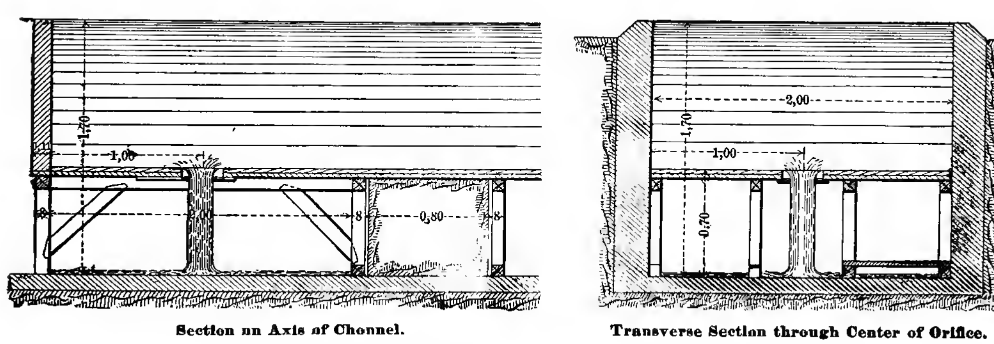
Abscissas or Distances from Point o.	Ordinates.				
	h = 792.	h = 836.	h = 885.	h = 949.	h = 1004.
0	179.6	178.3	178.0	178.6	179.7
10	175.1	171.9	172.4	174.0	172.5
20	170.3	167.9	168.1	168.2	168.9
30	165.8	166.4	166.4	165.8	166.9
40	162.9	162.8	162.5	163.7	164.1
50	162.2	161.8	161.3	160.2	162.9
60	160.6	159.7	157.9	159.4	161.7
70	158.1	158.7	157.7	157.7	158.2
80	157.1	155.5	156.2	156.0	156.8
90	155.0	153.1	153.1	156.6	156.4
100	154.6	153.1	151.6	152.5	154.6
110	152.5	151.4	150.6	150.6	152.4
120	149.3	150.4	149.6	150.1	152.4
130	148.8	148.6	149.9	151.0	151.1
140	148.0	147.3	147.6	148.6	149.8
150	146.5	146.5	145.6	148.4	148.4
160	144.2	145.1	144.1	146.0	145.3
170	142.6	143.6	143.6	141.6	142.8
180	140.7	142.5	138.6	141.4	142.6
190	140.5	138.8	138.1	139.6	142.8
200	137.3	139.1	.....	139.1	.....
210	136.5	137.6	136.0	139.6	141.1
230	132.3	133.6	132.0	139.6	135.6
250	130.5	127.6	131.1	133.3	135.2
270	127.4	126.5	126.4	127.8	131.8
290	122.6	123.8	123.1	126.6	126.7
310	.....	113.6	.....	120.3	124.2
320	115.1	.....	115.3	.....	.....
330	.....	112.7	.....	116.7	123.4
350	103.7	.....	109.6	.....	118.2
360	.....	106.2	.....	113.6	.....
370	.....	.....	.....	.....	111.9
390	93.7	95.7	98.6	106.8	106.3
420	.....	.....	.....	102.3	102.8
430	.....	87.8	89.2	.....	.....
440	82.8	.....	.....	.....	.....
450	.....	.....	.....	92.8	94.9
470	.....	.....	76.8	.....	.....
490	67.8	68.9	.....	80.9	81.9
510	.....	.....	65.3	.....	.....
530	.....	.....	.....	68.9	73.2
540	48.2	55.9	.....	.....	.....
550	.....	.....	52.9	.....	.....
590	31.4	34.0	38.1	52.7	56.0
630	.....	.....	.....	32.0	.....
640	.....	.....	17.0	.....	36.9
690	− 16.1	− 8.0	− 1.0	13.5	19.5
790	− 55.5	− 50.9	− 43.9	− 25.9	− 26.2
890	− 113.0	− 97.8	− 95.9	− 71.9	− 61.9
990	− 176.9	− 158.6	− 157.0	− 132.7	− 110.7
1090	.....	.....	− 219.7	− 190.7	− 166.6



BARRIER CONTAINING SQUARE OR CIRCULAR ORIFICE. SCALE 0.02 = 1.



BARRIER CONTAINING RECTANGULAR ORIFICE. SCALE 0.02 = 1.



ARRANGEMENT OF HORIZONTAL CIRCULAR ORIFICE. SCALE 0.02 = 1.

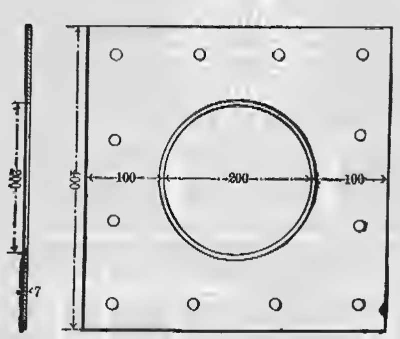
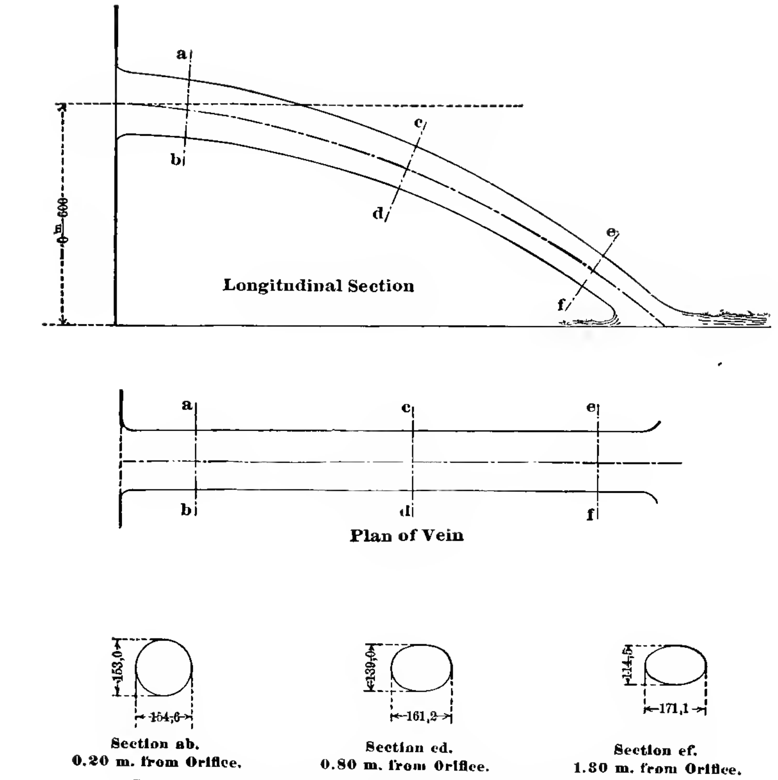
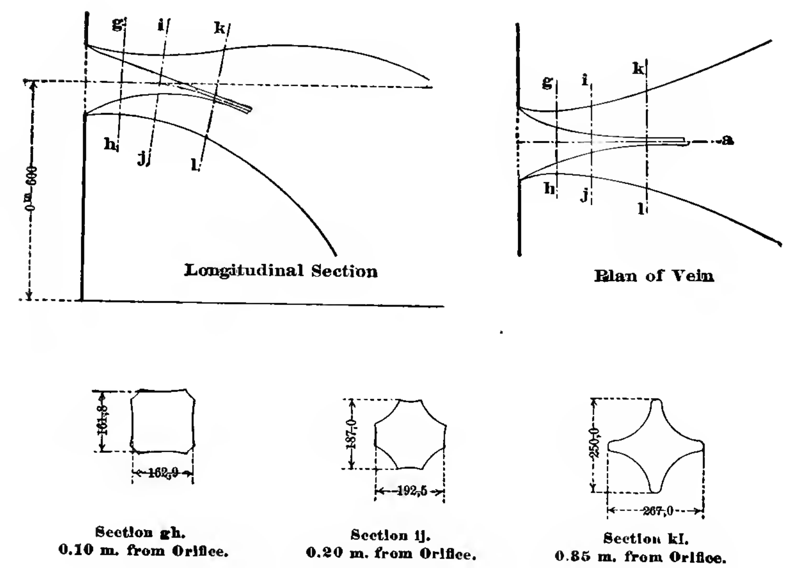


PLATE CONTAINING CIRCULAR ORIFICE.



VEIN ISSUING FROM VERTICAL CIRCULAR ORIFICE.
SCALE 0.05 = 1.



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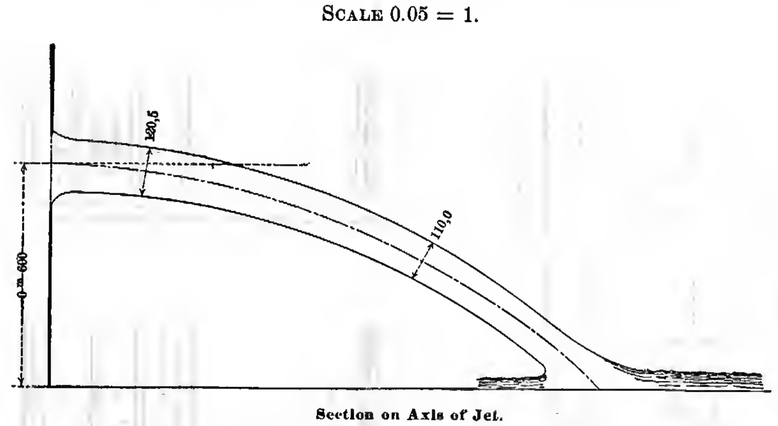

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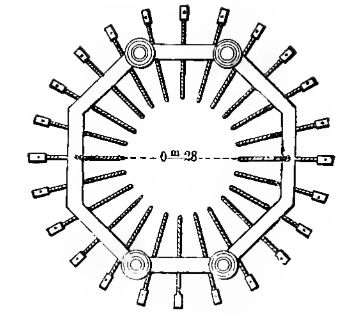

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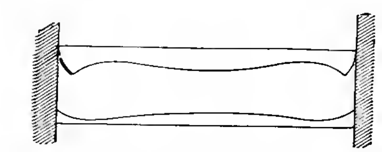

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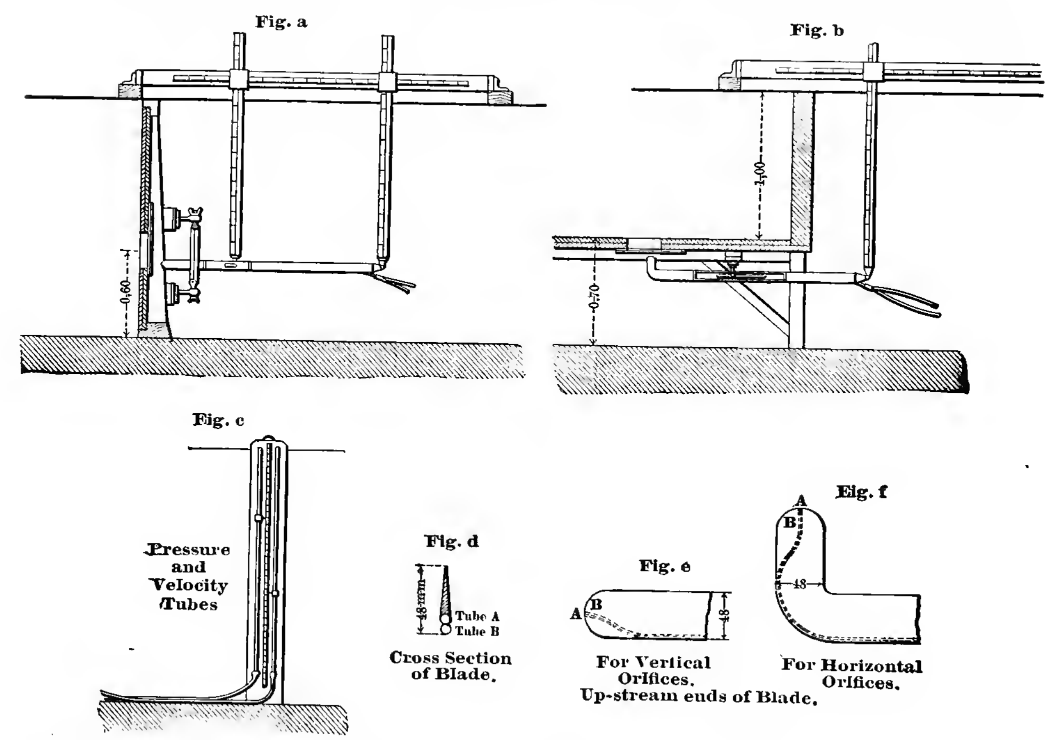

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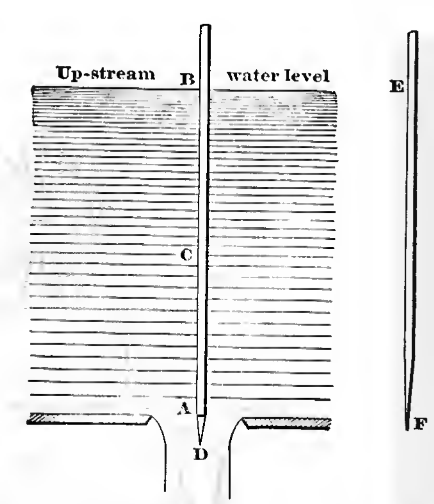

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